Zadatak 6. Kolike su duljine radijvektora dane točke T na danoj elipsi:

1) $T(2,y)$, $3x^2+4y^2=48$;

2) $T(\sqrt{6},y)$, $x^2+3y^2=12$;

3) $T(-1,y)$, $5x^2+9y^2=45$;

4) $T\left(\frac{12}{5},-4\right)$, $25x^2+16y^2=400$?

Rješenje. **1)**

$$T(2,y)$$
$$\underline{3x^2+4y^2=48}$$
$$r_1,\ \ r_2=?$$
$$3x^2+4y^2=48 \quad \big/ :48$$
$$\frac{x^2}{16}+\frac{y^2}{12}=1$$
$$\Longrightarrow a^2=16 \Longrightarrow a=4$$
$$b^2=12 \Longrightarrow b=2\sqrt{3}$$
$$e^2=a^2-b^2=4 \Longrightarrow e=2$$
$$F_1(-e,0),\ \ F_2(e,0) \Longrightarrow F_1(-2,0),\ \ F_2(2,0)$$
$$r_1+r_2=2a=8$$
$$r_1=d(T,F_1)$$
$$r_2=d(T,F_2)$$
$$T(2,y)\in e \Longrightarrow 3\cdot 2^2+4y^2=48$$
$$4y^2=48-12$$
$$4y^2=36$$
$$y^2=9$$
$$y=\pm 3 \Longrightarrow T_{1,2}(2,\pm 3)$$

Za T_1 i T_2 je r_1+r_2 jednako pa vrijedi

$$r_1=d(T_1,F_1)=\sqrt{(x_{T_1}-x_{F_1})^2+(y_{T_1}-y_{F_1})^2}=\sqrt{(2+2)^2+(3-0)^2}=5$$
$$r_2=d(T_1,F_2)=\sqrt{(x_{T_1}-x_{F_2})^2+(y_{T_1}-y_{F_2})^2}=\sqrt{(2-2)^2+(3-0)^2}=3$$

2)

$$T(\sqrt{6},y)$$
$$\underline{x^2+3y^2=12}$$
$$r_1,\ \ r_2=?$$
$$x^2+3y^2=12 \quad \big/ :12$$
$$\frac{x^2}{12}+\frac{y^2}{4}=1$$
$$\Longrightarrow a^2=12 \Longrightarrow a=2\sqrt{3}$$

$$b^2 = 4 \implies b = 2$$
$$e^2 = a^2 - b^2 = 8 \implies e = 2\sqrt{2}$$
$$F_1(-2\sqrt{2}, 0), \quad F_2(2\sqrt{2}, 0)$$
$$r_1 + r_2 = 2a = 4\sqrt{3}$$
$$r_1 = d(T, F_1)$$
$$r_2 = d(T, F_2)$$
$$T(\sqrt{6}, y) \in e \implies (\sqrt{6})^2 + 3y^2 = 12$$
$$3y^2 = 12 - 6$$
$$3y^2 = 6$$
$$y^2 = 2$$
$$y = \pm\sqrt{2} \implies T_{1,2}(\sqrt{6}, \pm\sqrt{2})$$

Za T_1 i T_2 je $r_1 + r_2$ jednako pa vrijedi

$$r_1 = d(T_1, F_1) = \sqrt{(-2\sqrt{2} - \sqrt{6})^2 + (0 - \sqrt{2})^2} = \sqrt{8 + 4\sqrt{12} + 6 + 2}$$
$$= \sqrt{16 + 8\sqrt{2}} = 2\sqrt{4 + 2\sqrt{2}} = 2\sqrt{(1 + \sqrt{3})^2} = 2(1 + \sqrt{3})$$
$$r_2 = d(T_1, F_2) = \sqrt{(2\sqrt{2} - \sqrt{6})^2 + (0 - \sqrt{2})^2} = \sqrt{8 - 4\sqrt{12} + 6 + 2}$$
$$= \sqrt{16 - 8\sqrt{2}} = 2\sqrt{4 - 2\sqrt{2}} = 2\sqrt{(1 - \sqrt{3})^2} = 2(\sqrt{3} - 1)$$

3)

$$T(-1, y)$$
$$5x^2 + 9y^2 = 45$$
$$r_1, \quad r_2 = ?$$
$$5x^2 + 9y^2 = 45 \quad / : 45$$
$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$
$$\implies a^2 = 9 \implies a = 3$$
$$b^2 = 5 \implies b = \sqrt{5}$$
$$e^2 = a^2 - b^2 = 4 \implies e = 2$$
$$F_1(-2, 0), \quad F_2(2, 0)$$
$$T(-1, y) \in e \implies 5 \cdot (-1)^2 + 9y^2 = 45$$
$$9y^2 = 40$$
$$y^2 = \frac{40}{9} \quad /\sqrt{\ }$$
$$y = \pm\frac{2\sqrt{10}}{3} \implies T_{1,2}\left(-1, \pm\frac{2\sqrt{10}}{3}\right)$$

$$r_1 = d(T_1, F_1) = \sqrt{(-1+2)^2 + \left(0 - \frac{2\sqrt{10}}{3}\right)^2} = \sqrt{1 + \frac{40}{9}} = \frac{7}{3}$$

$$r_2 = d(T_1, F_2) = \sqrt{(-1-2)^2 + \left(0 - \frac{2\sqrt{10}}{3}\right)^2} = \sqrt{9 + \frac{40}{9}} = \frac{11}{3}$$

4)

$$T\left(\tfrac{12}{5}, -4\right)$$

$$\underline{25x^2 + 16y^2 = 400}$$

$$r_1, \quad r_2 = ?$$

$$25x^2 + 16y^2 = 400 \quad / : 400$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

$$\Longrightarrow a^2 = 16 \Longrightarrow a = 4$$

$$b^2 = 25 \Longrightarrow b = 5$$

$$a < b \Longrightarrow e^2 = b^2 - a^2 = 9 \Longrightarrow e = 3$$

$$F_1(0, -3), \quad F_2(0, 3)$$

$$r_1 = d(T_1, F_1) = \sqrt{\left(0 - \frac{12}{5}\right)^2 + (-3+4)^2} = \frac{13}{5}$$

$$r_2 = d(T_1, F_2) = \sqrt{\left(0 - \frac{12}{5}\right)^2 + (3+4)^2} = \frac{37}{5}$$