Zadatak 7. Odredi jednadžbu elipse $b^2x^2 + a^2y^2 = a^2b^2$ ako su dane koordinate dviju točaka, A i B koje leže na elipsi:

1) $A(6,4)$, $B(-8,3)$; **2)** $A(9,4)$, $B(12,3)$;

3) $A(2,3)$, $B(-1,-5)$;

4) $A(-3,1)$, $B(2,-4)$.

Rješenje. **1)**

$$A(6,4)$$
$$B(-8,3)$$
$$\text{elipsa} \ \ldots \ b^2x^2 + a^2y^2 = a^2b^2$$
$$\left.\begin{array}{l} A \ \ldots \ 36b^2 + 16a^2 = a^2b^2 \quad (*) \\ B \ \ldots \ \underline{64b^2 + 9a^2 = a^2b^2} \end{array}\right\} -$$
$$-28b^2 + 7a^2 = 0 \quad / : 7$$
$$a^2 = 4b^2 \ \text{ uvrstimo u } (*)$$
$$36b^2 + 16 \cdot 4b^2 = 4b^2 \cdot b^2$$
$$36b^2 + 64b^2 = 4b^4$$
$$100b^2 = 4b^4 \quad / : 4b^2$$
$$b^2 = 25 \implies \boxed{b = 5}$$
$$a^2 = 4 \cdot 25 = 100 \implies \boxed{a = 10}$$
$$25x^2 + 100y^2 = 2500 \quad / : 25$$
$$\boxed{x^2 + 4y^2 = 100 \ \ldots \ E}$$

2)

$$A(9,4)$$
$$B(12,3)$$
$$\text{elipsa} \ \ldots \ b^2x^2 + a^2y^2 = a^2b^2$$
$$\left.\begin{array}{l} A \ \ldots \ 81b^2 + 16a^2 = a^2b^2 \quad (*) \\ B \ \ldots \ \underline{144b^2 + 9a^2 = a^2b^2} \end{array}\right\} -$$
$$-63b^2 + 7a^2 = 0 \quad / : 7$$
$$a^2 = 9b^2 \ \text{ uvrstimo u } (*)$$
$$81b^2 + 16 \cdot 9b^2 = 9b^2 \cdot b^2$$
$$81b^2 + 144b^2 = 9b^4$$
$$255b^2 = 9b^4 \quad / : 9b^2$$
$$b^2 = 25 \implies \boxed{b = 5}$$
$$a^2 = 9 \cdot 25 = 225 \implies \boxed{a = 15}$$
$$25x^2 + 225y^2 = 25 \cdot 225 \quad / : 25$$
$$\boxed{x^2 + 9y^2 = 225 \ \ldots \ E}$$

3)

$$A(2,3)$$
$$B(-1,-5)$$
$$\text{elipsa} \ \ldots \ b^2x^2 + a^2y^2 = a^2b^2$$
$$\left.\begin{array}{ll} A \ \ldots & 4b^2 + 9a^2 = a^2b^2 \quad (*) \\ B \ \ldots & \underline{b^2 + 25a^2 = a^2b^2} \end{array}\right\} -$$
$$3b^2 - 16a^2 = 0$$
$$b^2 = \frac{16}{3}a^2 \ \text{uvrstimo u} \ (*)$$
$$4 \cdot \frac{16}{3}a^2 + 9a^2 = a^2 \cdot \frac{16}{3}a^2$$
$$\frac{64}{3}a^2 + 9a^2 = \frac{16}{3}a^4$$
$$\frac{91}{3}a^2 = \frac{16}{3}a^4 \quad \Big/ \cdot \frac{3}{a^2}$$
$$91 = 16a^2$$
$$a^2 = \frac{91}{16} \implies b^2 = \frac{16}{3}a^2 = \frac{16}{3} \cdot \frac{91}{16} = \frac{91}{3}$$
$$\frac{91}{3}x^2 + \frac{91}{16}y^2 = \frac{91}{16} \cdot \frac{91}{3} \quad \Big/ \cdot \frac{3 \cdot 16}{91}$$
$$16x^2 + 3y^2 = 91 \ \ldots \ E$$

4)

$$A(-3,1)$$
$$B(2,-4)$$
$$\text{elipsa} \ \ldots \ b^2x^2 + a^2y^2 = a^2b^2$$
$$\left.\begin{array}{ll} A \ \ldots & 9b^2 + a^2 = a^2b^2 \quad (*) \\ B \ \ldots & \underline{4b^2 + 16a^2 = a^2b^2} \end{array}\right\} -$$
$$5b^2 - 15a^2 = 0 \quad \Big/ : 5$$
$$b^2 = 3a^2 \ \text{uvrstimo u} \ (*)$$
$$9 \cdot 3a^2 + a^2 = a^2 \cdot 3a^2$$
$$28a^2 = 3a^4 \quad \Big/ : a^2$$
$$a^2 = \frac{28}{3} \implies b^2 = 3 \cdot \frac{28}{3} = 28$$
$$28x^2 + \frac{28}{3}y^2 = 28 \cdot \frac{28}{3} \quad \Big/ \cdot \frac{3}{28}$$
$$3x^2 + y^2 = 28 \ \ldots \ E$$