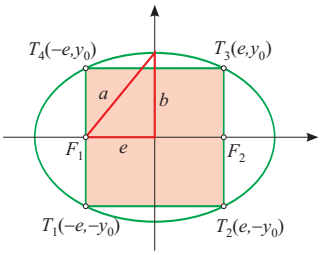
Zadatak 16. Stranica kvadrata upisanog elipsi prolazi njezinim žarištem. Koliki je numerički ekscentricitet elipse?

Rješenje.



$$b^2x^2 + a^2y^2 = a^2b^2$$
$$b^2e^2 + a^2y_0^2 = a^2b^2$$
$$a^2y_0^2 = a^2b^2 - b^2e^2$$
$$a^2y_0^2 = b^2(a^2 - e^2)$$
$$a^2y_0^2 = b^2[a^2 - (a^2 - b^2)]$$
$$a^2y_0^2 = b^4$$
$$y_0^2 = \frac{b^4}{a^2} \implies \boxed{y_0 = \frac{b^2}{a}}$$
$$2e = 2y_0 \text{ jer je } T_1T_2T_3T_4 \text{ kvadrat} \implies e = y_0$$
$$\varepsilon = \frac{e}{a} = \frac{\frac{b^2}{a}}{a} \implies \boxed{\varepsilon = \frac{b^2}{a^2}} \quad (*)$$
$$e = y_0 \quad /^2$$
$$a^2 - b^2 = \frac{b^4}{a^2} \quad / \cdot a^2$$
$$a^4 - b^2a^2 - b^4 = 0$$
$$(a^2)_{1,2} = \frac{b^2 \pm \sqrt{b^4 + 4b^4}}{2} = \frac{b^2 \pm b^2\sqrt{5}}{2} = \frac{b^2(1 \pm \sqrt{5})}{2}$$
$$\implies \boxed{a^2 = \frac{b^2(1+\sqrt{5})}{2}}$$
$$\text{vratimo se u } (*) \quad \ldots \quad \varepsilon = \frac{b^2}{\frac{b^2(1+\sqrt{5})}{2}} = \frac{2}{1+\sqrt{5}} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{2(1-\sqrt{5})}{1-5}$$
$$= \frac{2(1-\sqrt{5})}{-4} = \frac{\sqrt{5}-1}{2}$$