Zadatak 24. Udaljenost točke $T(8,12)$ elipse $b^2x^2 + a^2y^2 = a^2b^2$ od njezinog desnog žarišta jednaka je 12. Odredi jednadžbu elipse.

Rješenje.

$$T(8,12)$$
$$\underline{d(T,F_2) = 12}$$
$$b^2 \cdot 64 + a^2 \cdot 144 = a^2b^2 \quad / : a^2b^2$$
$$\frac{64}{a^2} + \frac{144}{b^2} = 1 \qquad (*)$$
$$F_2(e,0) \implies d = 12$$
$$\sqrt{(8-e)^2 + (0-12)^2} = 12 \quad /^2$$
$$(8-e)^2 + 144 = 144$$
$$(8-e)^2 = 0 \quad /\sqrt{\ }$$
$$\boxed{e = 8}$$
$$e^2 = a^2 - b^2$$
$$64 = a^2 - b^2$$
$$a^2 = b^2 + 64$$
$$\text{uvrstimo u } (*) \ \ldots \ \frac{64}{64+b^2} + \frac{144}{b^2} = 1 \quad / \cdot b^2(b^2+64)$$
$$64b^2 + 9216 + 144b^2 = b^4 + 64b^2$$
$$b^4 - 144b^2 - 9216 = 0$$
$$(b^2)_{1,2} = \frac{144 \pm \sqrt{144^2 + 4 \cdot 9216}}{2} = \frac{144 \pm 240}{2}$$
$$b_1^2 = \frac{144-240}{2} = -96 \ \text{(nije rješenje)}$$
$$b^2 = b_2^2 = \frac{144+240}{2} = 192$$
$$a^2 = 192 + 64 = 256$$
$$E \implies \frac{x^2}{256} + \frac{y^2}{192} = 1 \quad / \cdot 64 \cdot 4 \cdot 3$$
$$\boxed{3x^2 + 4y^2 = 768}$$