

**Zadatak 25.** Točka  $T(-4, 1)$  elipse  $b^2x^2 + a^2y^2 = a^2b^2$  udaljena je  $\sqrt{2}$  od njezinog lijevog žarišta. Odredi jednadžbu elipse.

*Rješenje.*

$$T(-4, 1)$$

$$d(T, F_1) = \sqrt{2}$$

$$F_1(-e, 0)$$

$$\{T\} \in E \implies b^2 \cdot 16 + a^2 \cdot 1 = a^2b^2$$

$$d(T, F_1) = \sqrt{2}$$

$$\sqrt{(-4+e)^2 + (1-0)^2} = \sqrt{2} \quad /^2$$

$$(-4+e)^2 + 1 = 2$$

$$(-4+e)^2 = 1 \quad / \sqrt{\phantom{x}}$$

$$1) \quad -4 + e = -1$$

$$e = 3$$

$$e^2 = a^2 - b^2$$

$$9 = a^2 - b^2$$

$$a^2 = 9 + b^2$$

$$16b^2 + 9 + b^2 = (9 + b^2)b^2$$

$$17b^2 + 9 - 9b^2 - b^4 = 0$$

$$b^4 - 8b^2 - 9 = 0$$

$$b_{1,2}^2 = \frac{8 \pm \sqrt{64 + 4 \cdot 9}}{2} = \frac{8 \pm \sqrt{100}}{2} = \frac{8 \pm 10}{2}$$

$$b_1^2 = \frac{8 - 10}{2} = -1 \quad (\text{nije rješenje})$$

$$b^2 = b_2^2 = \frac{8 + 10}{2} = 9$$

$$a^2 = 9 + 9 = 18$$

$$E_1 \quad \dots \quad \frac{x^2}{18} + \frac{y^2}{9} = 1 \quad / \cdot 18$$

$$x^2 + 2y^2 = 18$$

$$2) \quad -4 + e = 1$$

$$e = 5$$

$$e^2 = a^2 - b^2$$

$$25 = a^2 - b^2$$

$$a^2 = 25 + b^2$$

$$16b^2 + 25 + b^2 = (25 + b^2)b^2$$

$$17b^2 + 25 - 25b^2 - b^4 = 0$$

$$b^4 + 8b^2 - 25 = 0$$

$$b_{1,2}^2 = \frac{-8 \pm \sqrt{164}}{2} = \frac{-8 \pm 2\sqrt{41}}{2} = -4 \pm \sqrt{41}$$

$$b_1^2 = -4 - \sqrt{41} \text{ (nije rješenje)}$$

$$b^2 = b_2^2 = -4 + \sqrt{41}$$

$$a^2 = 25 - 4 + \sqrt{41} = 21 + \sqrt{41}$$

$$E_2 \quad \dots \quad \frac{x^2}{21 + \sqrt{41}} + \frac{y^2}{-4 + \sqrt{41}} = 1$$