Zadatak 26. Koja je točka elipse $2x^2 + 3y^2 = 24$ od njezinog lijevog žarišta udaljena $\sqrt{3}$?

Rješenje.

$$2x^2 + 3y^2 = 24$$
$$F_1(-e, 0)$$
$$\underline{d(T, F_1) = \sqrt{3}}$$
$$T(x_T, y_T) = ?$$
$$2x^2 + 3y^2 = 24 \quad / : 24$$
$$\frac{x^2}{12} + \frac{y^2}{8} = 1 \implies \boxed{a^2 = 12, \ b^2 = 8}$$
$$e^2 = a^2 - b^2 = 12 - 8 = 4 \implies \boxed{e = 2}$$
$$T(x_T, y_T)$$
$$d(T, F_1) = \sqrt{3}$$
$$\sqrt{(x_T + 2)^2 + (y_T - 0)^2} = \sqrt{3} \quad /^2$$
$$(x_T + 2)^2 + y_T^2 = 3 \qquad (*)$$
$$T \in E \implies 2x_T^2 + 3y_T^2 = 24$$
$$3y_T^2 = 24 - 2x_T^2$$
$$y_T^2 = 6 - \frac{2}{3}x_T^2$$

Uvrstimo u $(*)$:

$$x_T^2 + 4x_T + 4 + \left(8 - \frac{2}{3}x_T^2\right) = 3$$
$$\frac{1}{3}x_T^2 + 4x_T + 12 - 3 = 0 \quad / \cdot 3$$
$$x_T^2 + 12x_T + 27 = 0$$
$$(x_T)_{1,2} = \frac{-12 \pm 6}{2}$$
$$(x_T)_1 = -3$$
$$(x_T)_2 = -9$$
$$-a \leqslant x_T \leqslant a$$
$$-\sqrt{12} \leqslant x_T \leqslant \sqrt{12}$$
$$-3.5 \leqslant x_T \leqslant 3.5 \implies x_T = -9 \text{ nije rješenje}$$
$$\implies \boxed{x_T = -3}$$
$$y_T^2 = 8 - \frac{2}{3} \cdot 9 = 8 - 6 = 2$$
$$\boxed{y_T = \pm\sqrt{2}} \implies \boxed{T(-3, \pm\sqrt{2})}$$