

Zadatak 27. Koja je točka elipse $\frac{x^2}{100} + \frac{y^2}{36} = 1$ od njezinog desnog žarišta udaljena 14?

Rješenje.

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

$$F_2(e, 0)$$

$$d(T, F_2) = 14$$

$$T(x_T, y_T) = ?$$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1 \implies a^2 = 100, b^2 = 36$$

$$e^2 = a^2 - b^2 = 64 \implies e = 8$$

$$T \in E \implies \frac{x_T^2}{100} + \frac{y_T^2}{36} = 1$$

$$y_T^2 = \left(1 - \frac{x_T^2}{100}\right) \cdot 36 = 36 - \frac{9x_T^2}{25}$$

$$y_T = \sqrt{36 - \frac{9x_T^2}{25}}$$

$$d(T, F_2) = 14$$

$$\sqrt{(x_T - 8)^2 + (y_T - 0)^2} = 14$$

$$\sqrt{(x_T - 8)^2 + 36 - \frac{9x_T^2}{25}} = 14 \quad / \quad ^2$$

$$x_T^2 - 16x_T + 64 + 36 - \frac{9x_T^2}{25} - 196 = 0$$

$$\frac{16x_T^2}{25} - 16x_T - 96 = 0 \quad / \cdot \frac{25}{16}$$

$$x_T^2 - 25x_T - 150 = 0$$

$$(x_T)_{1,2} = \frac{25 \pm 35}{2}$$

$$(x_T)_1 = 35 \quad (\text{nije rješenje jer } x_T = 35 > a = 10)$$

$$x_T = -5$$

$$y_T^2 = \left(1 - \frac{25}{100}\right) \cdot 36 = 27$$

$$y_T = \pm 3\sqrt{3} \implies T(-5, \pm 3\sqrt{3})$$