Zadatak 28. Odredi točku na elipsi $5x^2+9y^2=405$ za koju je razlika udaljenosti od žarišta elipse jednaka 8.

Rješenje.

$$T(x_T, y_T)$$
$$F_1(-e,0)$$
$$F_2(e,0)$$
$$e^2=a^2-b^2=81-45=36$$
$$e=6 \implies F_1(-6,0),\ \ F_2(6,0)$$
$$r_1+r_2=2a$$
$$\underline{|r_1-r_2|=8}$$
$$r_1+r_2=18 \implies r_2=18-r_1$$
$$\underline{|r_1-r_2|=8}$$
$$|r_1-18+r_1|=8$$
$$|2r_1-18|=8$$

$$2r_1-18=-8 \qquad\qquad 2r_1-18=8$$
$$2r_1=10 \qquad\qquad 2r_1=26$$
$$\boxed{r_1=5} \qquad\qquad \boxed{r_1=13}$$

$$T(x_T, y_T)$$
$$5x_T^2+9y_T^2=405$$
$$9y_T^2=405-5x_T^2$$
$$y_T^2=45-\frac{5}{9}x_T^2$$

$\boxed{r_1=5}$ $\ldots$ $d(T,F_1)=5$

$$\sqrt{(x_T+6)^2+(y_T-0)^2}=5 \quad /^2$$
$$x_T^2+12x_T+36+y_T^2=25$$
$$x_T^2+12x_T+36+45-\frac{5}{9}x_T^2=25$$
$$\frac{4}{9}x_T^2+12x_T+56=0 \quad /\cdot 9$$
$$4x_T^2+108x_T+504=0 \quad /:4$$
$$x_T^2+27x_T+126=0$$
$$(x_T)_{1,2}=\frac{-27\pm 15}{2}$$
$$(x_T)_1=-21 \ \text{(nije rješenje jer } (x_T=-21<-a=-9)$$
$$x_T=-6 \implies y_T=\sqrt{45-\frac{5}{9}\cdot 36}=\sqrt{45-20}=\pm 5$$
$$\implies \boxed{T_{1,2}(-6,\pm 5)}$$

$r_1 = 13$ $\ldots$ $d(T, F_1) = 13$

$$\sqrt{(x_T + 6)^2 + (y_T - 0)^2} = 13 \quad /^2$$

$$x_T^2 + 12x_T + 36 + y_T^2 = 169$$

$$x_T^2 + 12x_T + 36 + 45 - \frac{5}{9}x_T^2 = 169$$

$$\frac{4}{9}x_T^2 + 12x_T - 88 = 0 \quad / \cdot \frac{9}{4}$$

$$x_T^2 + 27x_T - 198 = 0$$

$$(x_T)_{1,2} = \frac{-27 \pm \sqrt{729 + 792}}{2} = \frac{-27 \pm 39}{2}$$

$(x_T)_1 = -33$ (nije rješenje jer $(x_T = -33 < -a = -9)$

$$x_T = 6 \implies y_T = \sqrt{45 - \frac{5}{9} \cdot 36} = \sqrt{45 - 20} = \pm 5$$

$$\implies \quad T_{3,4}(6, \pm 5)$$