

Zadatak 30. Koliki kut zatvaraju radijvektori elipse $9x^2 + 25y^2 = 225$ koji pripadaju točki $T(3, y)$?

Rješenje.

$$9x^2 + 25y^2 = 225 \quad / : 225$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$e^2 = a^2 - b^2 = 25 - 9 = 16 \implies e = 4$$

$$F_1(-4, 0), \quad F_2(4, 0)$$

$$T(3, y) \dots 9 \cdot 9 + 25y^2 = 225$$

$$25y^2 = 144$$

$$y = \pm \frac{12}{5} \implies T_{1,2}\left(3, \pm \frac{12}{5}\right)$$

Izračunajmo koeficijent pravca F_1T_1 :

$$F_1(-4, 0)$$

$$T_1\left(3, -\frac{12}{5}\right)$$

$$k_1 = \frac{3 + 4}{-\frac{12}{5} - 0} = \frac{7}{-\frac{12}{5}} = -\frac{35}{12}$$

Izračunajmo još koeficijent pravca F_2T_1 :

$$F_2(4, 0)$$

$$T_1\left(3, -\frac{12}{5}\right)$$

$$k_2 = \frac{3 - 4}{-\frac{12}{5} + 0} = \frac{-1}{-\frac{12}{5}} = \frac{5}{12}$$

$$\operatorname{tg} \measuredangle(r_1, r_2) = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right| = \left| \frac{\frac{5}{12} + \frac{35}{12}}{1 - \frac{35}{12} \cdot \frac{5}{12}} \right| = \left| \frac{\frac{40}{12}}{-\frac{31}{144}} \right| = \left| -\frac{180}{31} \right| = \frac{480}{31}$$

$$\Rightarrow \measuredangle(r_1, r_2) = 86^\circ 19'$$