

Zadatak 36. Točka $T(2, 1)$ polovište je tetive elipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Kako glasi jednadžba pravca kojem pripada ta tetiva?

Rješenje.

$$E \dots \frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$T\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (2, 1)$$

$$\implies x_1 + x_2 = 4$$

$$y_1 + y_2 = 2$$

Točke $T_1(x_1, y_1)$ i $T_2(x_2, y_2)$ pripadaju elipsi pa je stoga

$$\frac{x_1^2}{25} + \frac{y_1^2}{16} = 1 \quad / \cdot 400$$

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$$16x_1^2 + 25y_1^2 = 400$$

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Oduzmemo li te jednadžbe, dobit ćemo

$$16(x_1^2 - x_2^2) + 25(y_1^2 - y_2^2) = 0$$

$$16(x_1 - x_2)(x_1 + x_2) + 25(y_1 - y_2)(y_1 + y_2) = 0$$

$$16(x_1 - x_2) \cdot 4 + 25(y_1 - y_2) \cdot 2 = 0 \quad / : 2$$

$$32(x_1 - x_2) + 25(y_1 - y_2) = 0 \quad / : 25(x_1 - x_2)$$

$$\frac{32}{25} + \frac{y_1 - y_2}{x_1 - x_2} = 0 \implies \frac{y_1 - y_2}{x_1 - x_2} = -\frac{32}{25}$$

$-\frac{32}{25}$ je koeficijent smjera pravca na kojem leži tetiva. Dakle

$$k = -\frac{32}{25}$$

$$T(2, 1) \in T_1T_2$$

$$T_1T_2 \dots y - 1 = -\frac{32}{25}(x - 2)$$

$$y - 1 = -\frac{32}{25}x + \frac{64}{25}$$

$$y + \frac{32}{25}x - \frac{89}{25} = 0 \quad / \cdot 25$$

$$32x + 25y - 89 = 0$$