Zadatak 37. Na elipsi $\frac{x^2}{20}+\frac{y^2}{4}=1$ odredi točke kojima su pripadni radijvektori međusobno okomiti.

Rješenje.

$$E \quad \dots \quad \frac{x^2}{20}+\frac{y^2}{4}=1$$
$$e=\sqrt{a^2-b^2}=\sqrt{20-4}=4$$
$$F_1(-4,0),\quad F_2(4,0)$$
$$r_1=d(F_1,T)$$
$$r_2=d(F_2,T)$$
$$F_1T\perp F_2T \implies k_1=-\frac{1}{k_2}$$
$$r_1+r_2=2a=2\sqrt{20}=2\cdot 2\sqrt{5}=4\sqrt{5}$$
$$T\in E \implies \frac{x_0^2}{20}+\frac{y_0^2}{4}=1 \quad /\cdot 20$$
$$x_0^2+5y_0^2=20$$
$$x_0^2=20-5y_0^2 \qquad (1)$$
$$k_1=k_{F_1T}=\frac{y_0-0}{x_0+4}=\frac{y_0}{x_0+4}$$
$$k_2=k_{F_2T}=\frac{y_0-0}{x_0-4}=\frac{y_0}{x_0-4}$$
$$k_1=-\frac{1}{k_2} \implies \frac{y_0}{x_0+4}=-\frac{1}{\frac{y_0}{x_0-4}}$$
$$\frac{y_0}{x_0+4}=-\frac{x_0-4}{y_0} \quad /\cdot -y_0(x_0+4)$$
$$-y_0^2=x_0^2-4^2 \implies x_0^2=16-y_0^2 \qquad (2)$$

Sada iz (1) i (2) imamo:

$$20-5y_0^2=16-y_0^2$$
$$4y_0^2=4$$
$$y_0^2=1 \quad /\sqrt{\ }$$
$$y_0=\pm 1$$
$$x_0^2=16-1=15$$
$$x_0=\pm\sqrt{15} \implies \boxed{T(\pm\sqrt{15},\pm 1)}$$