Zadatak 45. Elipsa $x^2 + 4y^2 = 25$ siječe koordinatne osi u točkama koje su vrhovi romba. Napiši jednadžbu kružnice upisane tom rombu.

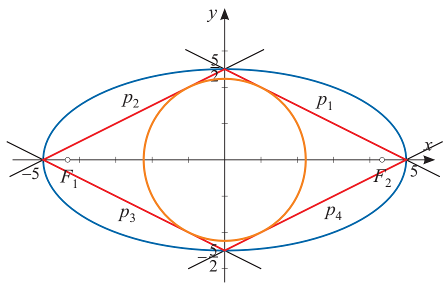
Rješenje.

$$x^2 + 4y^2 = 25 \quad / : 25$$

$$\frac{x^2}{25} + \frac{y^2}{\frac{25}{4}} = 1$$

$$\Longrightarrow \quad a = 5, \quad b = \frac{5}{2}$$

$$e = \sqrt{a^2 - b^2} = \sqrt{25 - \frac{25}{4}} = \sqrt{\frac{75}{4}} = \frac{5\sqrt{3}}{2}$$



p_1, p_2, p_3, p_4 su tangente na kružnicu k:

$$p_1 \quad \ldots \quad \frac{x}{m} + \frac{y}{n} = 1$$

$$\frac{x}{5} + \frac{y}{\frac{5}{2}} = 1 \quad / \cdot 5$$

$$x + 2y = 5$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

$$p_2 \quad \ldots \quad \frac{x}{-5} + \frac{y}{\frac{5}{2}} = 1 \quad / \cdot 5$$

$$-x + 2y = 5$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

$$p_3 \quad \ldots \quad p_3 \| p_1 \Longrightarrow \quad y = -\frac{1}{2}x - \frac{5}{2}$$

$$p_4 \quad \ldots \quad p_4 \| p_2 \Longrightarrow \quad y = \frac{1}{2}x - \frac{5}{2}$$

Uvjet tangente na kružnicu sa središtem u ishodištu:

$$l^2 = r^2(1 + k^2)$$

$$p_1 \quad \ldots \quad \left(\frac{5}{2}\right)^2 = r^2\left(1 + \frac{1}{4}\right)$$

$$\frac{25}{4} = r^2 \cdot \frac{5}{4}$$

$$r^2 = 5 \implies r = \sqrt{5}$$

$$k \quad \ldots \quad x^2 + y^2 = 5$$