

Zadatak 55. Grafički prikaži skup točaka ravnine zadan jednadžbom:

$$1) y = \frac{1}{3}\sqrt{36 - 4x^2}; \quad 2) y = -\frac{1}{2}\sqrt{4x - x^2};$$

$$3) y = -1 + \frac{2}{5}\sqrt{16 - 6x - x^2};$$

$$4) y = 1 - \frac{4}{3}\sqrt{6x - x^2}.$$

Rješenje.

$$1) 36 - 4x^2 \geq 0$$

$$4x^2 \leq 36$$

$$x^2 \leq 9$$

$$-3 \leq x \leq 3$$

$$y = \frac{1}{3}\sqrt{36 - 4x^2} \implies y \geq 0$$

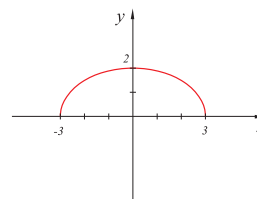
$$y = \frac{1}{3}\sqrt{36 - 4x^2} \quad / \cdot 3$$

$$3y = \sqrt{36 - 4x^2} \quad /^2$$

$$9y^2 = 36 - 4x^2$$

$$4x^2 + 9y^2 = 36 \quad / : 36$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \quad (\text{dio elipse } (a = 3, b = 2) \text{ za koji je } y \geq 0)$$



$$2) 4x - x^2 \geq 0$$

$$x(4 - x) \geq 0$$

$$x \in [0, 4]$$

$$y = -\frac{1}{2}\sqrt{4x - x^2} \implies y \leq 0$$

$$y = -\frac{1}{2}\sqrt{4x - x^2} \quad / \cdot (-2)$$

$$-2y = \sqrt{4x - x^2} \quad /^2$$

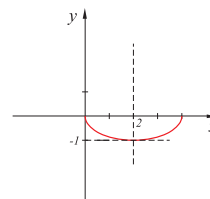
$$4y^2 = 4x - x^2$$

$$x^2 - 4x + 4y^2 = 0$$

$$(x - 2)^2 - 4 + 4y^2 = 0$$

$$(x - 2)^2 + 4y^2 = 4 \quad / : 4$$

$$\frac{(x - 2)^2}{4} + y^2 = 1 \quad \text{dio elipse } (C(2, 0), a = 2, b = 1) \text{ za koji je } y \leq 0$$



$$3) \quad 16 - 6x - x^2 \geq 0 \quad / \cdot (-1)$$

$$x^2 + 6x - 16 \leq 0$$

$$x_{1,2} = \frac{-6 \pm \sqrt{36 + 64}}{2} = \frac{-6 \pm 10}{2}, \quad x_1 = -8, x_2 = 2 \implies x \in [-8, 2]$$

$$y = -1 + \frac{2}{5} \sqrt{16 - 6x - x^2}$$

$$y + 1 = \underbrace{\frac{2}{5} \sqrt{16 - 6x - x^2}}_{\geq 0} \implies y + 1 \geq 0, \quad y \geq -1$$

$$y + 1 = \frac{2}{5} \sqrt{16 - 6x - x^2} \quad / \cdot \frac{5}{2}$$

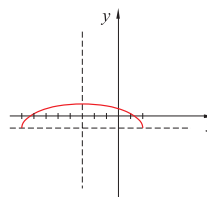
$$\frac{5}{2}(y + 1) = \sqrt{16 - 6x - x^2} \quad /^2$$

$$\frac{25}{4}(y + 1)^2 = 16 - 6x - x^2$$

$$\frac{25}{4}(y + 1)^2 = -(x + 3)^2 + 9 + 16$$

$$(x + 3)^2 + \frac{25}{4}(y + 1)^2 = 25 \quad / : 25$$

$$\frac{(x + 3)^2}{25} + \frac{(y + 1)^2}{4} = 1 \quad \text{dio elipse } (C(-3, -1), a = 5, b = 2) \text{ za koji je } y \geq -1$$



$$4) \quad 6x - x^2 \geq 0$$

$$x(6 - x) \geq 0 \implies x \in [0, 6]$$

$$y = 1 - \frac{4}{3} \sqrt{6x - x^2}$$

$$y - 1 = \underbrace{-\frac{4}{3} \sqrt{6x - x^2}}_{\leq 0} \implies y - 1 \leq 0, \quad y \leq 1$$

$$y - 1 = -\frac{4}{3} \sqrt{6x - x^2} \quad / \cdot \frac{3}{4}$$

$$\frac{3}{4}(y - 1) = -\sqrt{6x - x^2} \quad /^2$$

$$\frac{9}{16}(y - 1)^2 = 6x - x^2$$

$$\frac{9}{16}(y - 1)^2 = -(x - 3)^2 + 9$$

$$(x - 3)^2 + \frac{9}{16}(y - 1)^2 = 9 \quad / : 9$$

$$\frac{(x - 3)^2}{9} + \frac{(y - 1)^2}{16} = 1 \quad \text{dio elipse } (C(3, 1), a = 3, b = 4) \text{ za koji je } y \leq 1$$

