

Zadatak 9. Kako glasi jednadžba hiperbole $b^2x^2 - a^2y^2 = a^2b^2$ koja prolazi točkama M i N :

- 1) $M(-3, \sqrt{15}), N(4, 6)$;
- 2) $M(2\sqrt{3}, 1), N(-4, -\sqrt{2})$;
- 3) $M\left(5, \frac{9}{4}\right), N\left(\frac{20}{3}, 4\right)$;
- 4) $M(-5, 1), N(7, -5)$?

Rješenje.

1)

$$M(-3, \sqrt{15})$$

$$N(4, 6)$$

$$b^2x^2 - a^2y^2 = a^2b^2$$

$$\left. \begin{array}{l} M \dots 9b^2 - 15a^2 = a^2b^2 \quad (1) \\ N \dots \underline{16b^2 - 36a^2 = a^2b^2} \end{array} \right\} -$$

$$-7b^2 + 21a^2 = 0$$

$$b^2 = 3a^2$$

$$(1) \dots 9 \cdot 3a^2 - 15a^2 = a^2 \cdot 3a^2$$

$$12a^2 = 3a^4 \quad / : 3a^2$$

$$a^2 = 4 \implies b^2 = 3 \cdot 4 = 12$$

$$H \dots 12x^2 - 4y^2 = 48 \quad / : 4$$

$$3x^2 - y^2 = 12$$

2)

$$M(2\sqrt{3}, 1)$$

$$N(-4, -\sqrt{2})$$

$$b^2x^2 - a^2y^2 = a^2b^2$$

$$\left. \begin{array}{l} M \dots 12b^2 - a^2 = a^2b^2 \quad (1) \\ N \dots \underline{16b^2 - 2a^2 = a^2b^2} \end{array} \right\} -$$

$$-4b^2 + a^2 = 0$$

$$a^2 = 4b^2$$

$$(1) \dots 12b^2 - 4b^2 = 4b^2 \cdot b^2$$

$$8b^2 = 4b^4 \quad / : 4b^2$$

$$b^2 = 2 \implies a^2 = 4 \cdot 2 = 8$$

$$H \dots 2x^2 - 8y^2 = 16 \quad / : 2$$

$$x^2 - 4y^2 = 8$$

3)

$$\begin{array}{l}
 M\left(5, \frac{9}{4}\right) \\
 N\left(\frac{20}{3}, 4\right) \\
 b^2x^2 - a^2y^2 = a^2b^2 \\
 \left. \begin{array}{l}
 M \dots 25b^2 - \frac{81}{16}a^2 = a^2b^2 \quad (1) \\
 N \dots \frac{400}{9}b^2 - 16a^2 = a^2b^2
 \end{array} \right\} - \\
 -\frac{175}{9}b^2 + \frac{175}{16}a^2 = 0 \\
 \frac{175}{16}a^2 = \frac{175}{9}b^2 \quad / \cdot \frac{16}{175} \\
 a^2 = \frac{16}{9}b^2 \\
 (1) \dots 25b^2 - \frac{81}{16} \cdot \frac{16}{9}b^2 = \frac{16}{9}b^2 \cdot b^2 \\
 25b^2 - 9b^2 = \frac{16}{9}b^4 \\
 16b^2 = \frac{16}{9}b^4 \quad / \cdot \frac{9}{16b^2} \\
 b^2 = 9 \implies a^2 = \frac{16}{9} \cdot 9 = 16
 \end{array}$$

$$H \dots 9x^2 - 16y^2 = 144$$

4)

$$\begin{array}{l}
 M(-5, 1) \\
 N(7, -5) \\
 b^2x^2 - a^2y^2 = a^2b^2 \\
 \left. \begin{array}{l}
 M \dots 25b^2 - a^2 = a^2b^2 \quad (1) \\
 N \dots 49b^2 - 25a^2 = a^2b^2
 \end{array} \right\} - \\
 -24b^2 + 24a^2 = 0 \\
 a^2 = b^2 \\
 (1) \dots 25a^2 - a^2 = a^2 \cdot a^2 \\
 24a^2 = a^4 \quad / : a^2 \\
 a^2 = 24 \implies b^2 = 24
 \end{array}$$

$$H \dots 24x^2 - 24y^2 = 24 \cdot 24 \quad / : 24$$

$$x^2 - y^2 = 24$$