

**Zadatak 15.** Asimptote hiperbole  $4x^2 - 9y^2 = 12$  su tangente kružnice čije je središte u žarištu iste hiperbole. Koliki je polumjer kružnice?

*Rješenje.*

$$4x^2 - 9y^2 = 12 \quad / : 12$$

$$\frac{x^2}{3} - \frac{y^2}{\frac{12}{9}} = 1$$

$$\frac{x^2}{3} - \frac{y^2}{\frac{4}{3}} = 1$$

$$\Rightarrow a^2 = 3, \quad a = \sqrt{3}$$

$$b^2 = \frac{4}{3}, \quad b = \frac{2}{\sqrt{3}}$$

$$e^2 = a^2 + b^2 = 3 + \frac{4}{3} = \frac{13}{3} \Rightarrow e = \sqrt{\frac{13}{3}}$$

Središte kružnice:

$$S_1\left(-\sqrt{\frac{13}{3}}, 0\right) \Rightarrow p = -\sqrt{\frac{13}{3}}, \quad q = 0$$

$$S_2\left(\sqrt{\frac{13}{3}}, 0\right) \Rightarrow p = \sqrt{\frac{13}{3}}, \quad q = 0$$

Asimptote hiperbole:

$$y = \pm \frac{b}{a}x = \pm \frac{\frac{2}{\sqrt{3}}}{\sqrt{3}}x = \pm \frac{2}{3}x$$

Tangente kružnice:

$t_1 \dots y = -\frac{2}{3}x$  je tangenta kružnice sa  $S\left(-\sqrt{\frac{13}{3}}, 0\right)$ ,  $k = -\frac{2}{3}$ ,  $l = 0$  pa imamo:

$$(-kp + q - l)^2 = r^2(1 + k^2)$$

$$\left(\frac{2}{3} \cdot \left(-\sqrt{\frac{13}{3}}\right) - 0 - 0\right)^2 = r^2\left(1 + \frac{4}{9}\right)$$

$$\left(-\frac{2\sqrt{13}}{3\sqrt{3}}\right)^2 = r^2 \frac{13}{9}$$

$$\frac{52}{27} = r^2 \frac{13}{9} \quad / \cdot \frac{9}{13}$$

$$r^2 = \frac{4}{3} \Rightarrow r = \frac{2}{\sqrt{3}}$$