Zadatak 17. Središte kružnice $(x+3)^2+y^2=4$ u žarištu je hiperbole, a kružnica dira asimptote hiperbole. Odredi jednadžbu hiperbole.

Rješenje.

$$(x+3)^2+y^2=4 \implies S(-3,0)=F_1(-3,0) \text{ (fokus hiperbole)}$$

$$y=\pm\frac{b}{a}x \text{ asimptote hiperbole} \equiv \text{tangente kružnice}$$

Uvjet za tangentu kružnice:

$$r^2(1+k^2)=(q-kp-l)^2$$

$$4\left(1+\frac{b^2}{a^2}\right)=\left(0\pm\frac{b}{a}\cdot -0\right)^2$$

$$4\left(1+\frac{b^2}{a^2}\right)=9\frac{b^2}{a^2}$$

$$4=5\frac{b^2}{a^2} \quad \Big/ \cdot\frac{a^2}{4}$$

$$\boxed{a^2=\frac{5}{4}b^2}$$

$$e=3 \implies e^2=a^2+b^2$$

$$9=\frac{5}{4}b^2+b^2$$

$$9=\frac{9}{4}b^2 \quad \Big/ \cdot\frac{4}{9}$$

$$\boxed{b^2=4}$$

$$a^2=\frac{5}{4}b^2=\frac{5}{4}\cdot 4=5 \implies \boxed{a^2=5}$$

$$\boxed{H} \quad \ldots \quad \boxed{\frac{x^2}{5}-\frac{y^2}{4}=1}$$