

**Zadatak 18.** Središte kružnice  $(x + 5)^2 + y^2 = 9$  žarište je hiperbole  $b^2x^2 - a^2y^2 = a^2b^2$ . Kružnica dira obje asimptote hiperbole. Kako glasi jednačba hiperbole?

**Rješenje.**

$$(x + 5)^2 + y^2 = 9 \implies S_k(-5, 0) = F_H(-5, 0)$$

$y = \pm \frac{b}{a}x$  asimptote hiperbole i tangente kružnice.

Uvjet da  $y = kx + l$  bude tangenta kružnice:

$$r^2(1 + k^2) = (q - kp - l)^2$$

$$9\left(1 + \frac{a^2}{b^2}\right) = \left(0 \pm \frac{b}{a} \cdot 5 - 0\right)^2$$

$$9 + \frac{9a^2}{b^2} = 25 \frac{b^2}{a^2}$$

$$9 = 16 \frac{b^2}{a^2} \quad / \cdot \frac{a^2}{16}$$

$$b^2 = \frac{9a^2}{16}$$

$$e^2 = a^2 + b^2$$

$$25 = a^2 + \frac{9a^2}{16}$$

$$25 = \frac{25a^2}{16} \quad / \cdot \frac{16}{25}$$

$$a^2 = 16 \implies b^2 = 9$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1 \quad / \cdot 144$$

$$9x^2 - 16y^2 = 144$$