

**Zadatak 21.** Kružnica  $x^2 + (y-5)^2 = 16$  dira asimptote hiperbole  $b^2x^2 - a^2y^2 = a^2b^2$ . Ako je linearni ekscentricitet hiperbole jednak 10, kako glasi njezina jednadžba?

*Rješenje.*

$$k \dots x^2 + (y-5)^2 = 16$$

$$H \dots b^2x^2 - a^2y^2 = a^2b^2, \quad e = 10$$

Tangente kružnice su asimptote hiperbole  $(y = \pm \frac{b}{a}x)$ .

Uvjet da  $y = kx + l$  bude tangenta kružnice:

$$r^2(1+k^2) = (q - kp - l)^2$$

$$16\left(1 + \frac{a^2}{b^2}\right) = \left(5 \pm \frac{b}{a} \cdot 0 - 0\right)^2$$

$$16 + \frac{16a^2}{b^2} = 25$$

$$16 \frac{b^2}{a^2} = 9 \quad / \cdot \frac{a^2}{16}$$

$$b^2 = \frac{9}{16}a^2$$

$$e^2 = a^2 + b^2$$

$$100 = a^2 + \frac{9}{16}a^2$$

$$100 = \frac{25}{16}a^2 \quad / \sqrt{\quad}$$

$$10 = \frac{5}{4}a \quad / \cdot \frac{4}{5}$$

$$a = 8$$

$$b^2 = \frac{9}{16} \cdot 64 = 36 \implies b = 6$$

$$H \dots \frac{x^2}{64} - \frac{y^2}{36} = 1 \quad / \cdot 576$$

$$9x^2 - 16y^2 = 576$$