Zadatak 26. Točka $(1,2)$ leži na hiperboli kojoj su pravci $y = \pm\frac{1}{2}x$ asimptote. Odredi jednadžbu hiperbole.

Rješenje.

$$T(1,2)$$

$$y = \pm 12x \ \text{(asimptota } y = \pm\frac{b}{a}x)$$

$$\Longrightarrow \frac{b}{a} = \frac{1}{2}, \qquad 2b = a$$

$$T(1,2) \in H \ \ldots \ b^2 - 4 \cdot 4b^2 = 4b^2 \cdot b^2$$

$$-15b^2 = 4b^4 \quad \text{nema rješenja}$$

Pogledajmo hiperbolu $-b^2x^2 + a^2y^2 = a^2b^2$:

$$-b^2 + 4a^2 = a^2b^2$$

$$-b^2 + 16b^2 = 4b^4$$

$$15b^2 = 4b^4 \quad / : 4b^2$$

$$b^2 = \frac{15}{4} \Longrightarrow \quad b = \frac{\sqrt{15}}{2}$$

$$a = 2 \cdot b = 2 \cdot \frac{\sqrt{15}}{2} \Longrightarrow \quad a = \sqrt{15}$$

$$H \ \ldots \ -\frac{15}{4}x^2 + 15y^2 = \frac{15}{4} \cdot 15 \quad / \cdot \frac{4}{15}$$

$$-x^2 + 4y^2 = 15$$