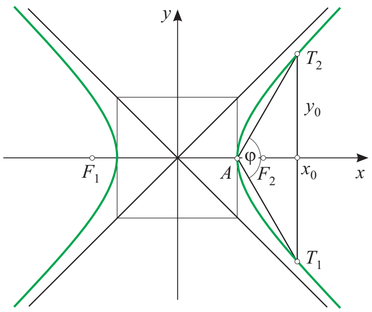
Zadatak 33. Hiperboli $x^2 - y^2 = a^2$ upisan je jednakokračan trokut kojem je vrh u tjemenu hiperbole, a kut pri tom vrhu $120°$. Kolika je površina tog trokuta?

Rješenje.



$$H \ \ldots \ x^2 - y^2 = a^2 \quad / : a^2$$

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

$$T_{1,2}(x_0, \pm y_0)$$

$$\operatorname{ctg}\frac{\varphi}{2} = \frac{x_0 - a}{y_0}$$

$$\operatorname{ctg} 60° = \frac{x_0 - a}{y_0}$$

$$\frac{\sqrt{3}}{3} = \frac{x_0 - a}{y_0}$$

$$x_0 - a = \frac{\sqrt{3}y_0}{3} \implies \boxed{x_0 = a + \frac{\sqrt{3}y_0}{3}}$$

$$x_0^2 - y_0^2 = a^2$$

$$\left(a + \frac{\sqrt{3}y_0}{3}\right)^2 - y_0^2 = a^2$$

$$a^2 + \frac{2\sqrt{3}}{3}ay_0 + \frac{3y_0^2}{9} - y_0^2 = a^2$$

$$\frac{2\sqrt{3}}{3}ay_0 - \frac{2}{3}y_0^2 = 0$$

$$\frac{2}{3}y_0(\sqrt{3}a - y_0) = 0$$

$y_0 = 0$ nije rješenje

$$\sqrt{3}a - y_0 = 0 \implies \boxed{y_0 = \sqrt{3}a}$$

$$x_0 = a + \frac{\sqrt{3}}{3} \cdot \sqrt{3}a \implies \boxed{x_0 = 2a}$$

$$P = \frac{2y_0 \cdot (x_0 - a)}{2} = y_0 \cdot (x_0 - a) = \sqrt{3}a \cdot (2a - a) = a^2\sqrt{3}$$

$$\implies \boxed{P = a^2\sqrt{3}}$$