Zadatak 36. Točka $T(5,1)$ polovište je tetive hiperbole $4x^2-9y^2=36$. Kojem pravcu pripada ta tetiva?

Rješenje.

$$T(5,1)$$

$$H \ \ldots \ 4x^2-9y^2=36$$

T je polovište tetive AB $(A(x_1,y_1),\ B(x_2,y_2))$:

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=(5,1)$$

$$\frac{x_1+x_2}{2}=5,\quad x_1+x_2=10 \implies x_2=10-x_1$$

$$\frac{y_1+y_2}{2}=1,\quad y_1+y_2=2 \implies y_2=2-y_1$$

$$\{A\}\in H \ \ldots \ 4x_1^2-9y_1^2=36$$

$$\{B\}\in H \ \ldots \ 4x_2^2-9y_2^2=36$$

$$4x_1^2-9y_1^2=36$$

$$4(10-x_1)^2-9(2-y_1)^2=36$$

$$4x_1^2-9y_1^2=36$$

$$4(100-20x_1+x_1^2)-9(4-4y_1+y_1^2)=36$$

$$4x_1^2-9y_1^2=36$$

$$400-80x_1+4x_1^2-36+36y_1-9y_1^2=36$$

$$\left.\begin{aligned}4x_1^2-9y_1^2&=36\\ 4x_1^2-9y_1^2-80x_1+36y_1&=-328\end{aligned}\right\}-$$

$$80x_1-36y_1=364 \quad /:4$$

$$20x_1-9y_1=91 \implies p \ \ldots \ 20x-9y-91=0$$