

Zadatak 37. Odredi jednadžbu pravca točkom $T(3, -1)$, tako da ta točka bude polovište odsječka što ga pravac odsijeca na hiperboli $7x^2 - 3y^2 = 21$.

Rješenje.

$$T(3, -1)$$

$$H \dots 7x^2 - 3y^2 = 21$$

T je polovište tetive AB ($A(x_1, y_1)$, $B(x_2, y_2)$):

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = (3, -1)$$

$$\frac{x_1 + x_2}{2} = 3, \quad x_1 + x_2 = 6 \implies x_2 = 6 - x_1$$

$$\frac{y_1 + y_2}{2} = -1, \quad y_1 + y_2 = -2 \implies y_2 = -2 - y_1$$

$$\{A\} \in H \dots 7x_1^2 - 3y_1^2 = 21$$

$$\{B\} \in H \dots 7x_2^2 - 3y_2^2 = 21$$

$$7x_1^2 - 3y_1^2 = 21$$

$$7(6 - x_1)^2 - 3(-2 - y_1)^2 = 21$$

$$7x_1^2 - 3y_1^2 = 21$$

$$7(36 - 12x_1 + x_1^2) - 3(4 + 4y_1 + y_1^2) = 21$$

$$7x_1^2 - 3y_1^2 = 21$$

$$252 - 84x_1 + 7x_1^2 - 12 - 12y_1 - 3y_1^2 = 21$$

$$7x_1^2 - 3y_1^2 = 21$$

$$7x_1^2 - 3y_1^2 - 84x_1 - 12y_1 = -219$$

$$84x_1 + 12y_1 = 240 \quad / : 12$$

$$7x_1 + y_1 = 20 \implies p \dots 7x + y - 20 = 0$$