Zadatak 44. Na hiperboli $-\frac{x^2}{36}+\frac{y^2}{64}=1$ odredi točku kojoj je udaljenost od jednog žarišta jednaka $\frac{9}{2}$.

Rješenje.

$$-\frac{x^2}{36}+\frac{y^2}{64}=1 \implies \boxed{a=6}, \quad \boxed{b=8}$$

$$e^2=a^2+b^2=36+64=100 \implies \boxed{e=10}$$

$$\boxed{F_{1,2}(0,\pm 10)}$$

$$T(x_0,y_0)$$

$$d(F_1,T)=\frac{9}{2}$$

$$\sqrt{(x_0-0)^2+(y_0+10)^2}=\frac{9}{2} \quad /^2$$

$$x_0^2+y_0^2+20y_0+100=\frac{81}{4}$$

$$x_0^2+y_0^2+20y_0=-\frac{319}{4} \qquad (1)$$

$$\{T\}\in H \implies -\frac{x_0^2}{36}+\frac{y_0^2}{64}=1 \quad /\cdot 576$$

$$(1) \implies x_0^2+y_0^2+20y_0=-\frac{319}{4} \quad /\cdot 16$$

$$\left.\begin{aligned} -16x_0^2+9y_0^2&=576\\ 16x_0^2+16y_0^2+320y_0&=-1276\end{aligned}\right\}+$$

$$25y_0^2+320y_0=-700 \quad /:5$$

$$5y_0^2+64y_0+140=0$$

$$(y_0)_{1,2}=\frac{-64\pm\sqrt{4096-2800}}{10}=\frac{-64\pm 36}{10}$$

$(y_0)_1=-2.8$ nije rješenje jer $-b\leqslant (y_0)_1\leqslant b$

$$\boxed{y_0=-10}$$

$$-16x_0^2+9\cdot 100=576$$

$$-16x_0^2=-324 \quad /:(-16)$$

$$x_0^2=\frac{324}{16}=\frac{81}{4} \implies \boxed{x_0=\pm\frac{9}{2}}$$

$T_{1,2}$ udaljena za $\frac{9}{2}$ od F_1 je $\boxed{T_{1,2}\left(-\frac{9}{2},\pm 10\right)}$,

$T_{3,4}$ udaljena za $\frac{9}{2}$ od F_2 je $\boxed{T_{1,2}\left(\frac{9}{2},\pm 10\right)}$.