Zadatak 37. Odredi jednadžbu pravca koji prolazi točkom $A(2,5)$ i siječe parabolu $y^2 = 20x$ tako da je A polovište tetive što je na pravcu odsijeca parabola.

Rješenje.

$$A(2,5)$$
$$P \quad \ldots \quad y^2 = 20x$$
$$p \quad \ldots \quad y = kx + l$$
$$\{A\} \in p \implies 5 = 2k + l, \quad l = 5 - 2k$$
$$p \quad \ldots \quad y = kx + 5 - 2k$$

Presjek pravca i parabole:

$$(kx + 5 - 2k)^2 = 20x$$
$$k^2x^2 + 25 + 4k^2 + 10kx - 4k^2x - 20k - 20x = 0$$
$$k^2x^2 + (10k - 4k^2 - 20)x + 25 - 20k - 4k^2 = 0$$

Pravac siječe parabolu u dvije točke pa kvadratna jednadžba ima dva rješenja, tj. mora vrijediti $D > 0$:

$$D > 0$$
$$(10k - 4k^2 - 20)^2 - 4k^2(25 - 20k - 4k^2) > 0$$
$$100k^2 + 16k^4 + 400 - 80k^3 - 400k + 160k^2 - 100k^2 + 80k^3 - 16k^4 > 0$$
$$160k^2 - 400k + 400 > 0 \quad \Big/ : 80$$
$$2k^2 - 5k + 5 > 0$$
$$k_{1,2} = \frac{5 \pm \sqrt{25 - 40}}{4}. \qquad k \in \mathbf{R}$$

A je polovište dužine $\overline{T_1T_2}$:

$$\frac{x_1 + x_2}{2} = 2, \quad \frac{y_1 + y_2}{2} = 5$$
$$x_{1,2} = \frac{-10k + 4k^2 + 20 \pm \sqrt{160k^2 - 400k + 400)}}{2k^2}$$
$$x_{1,2} = \frac{-10k + 4k^2 + 20 \pm \sqrt{16(10k^2 - 25k + 25)}}{2k^2}$$
$$x_{1,2} = \frac{-5k + 2k^2 + 10 \pm 2\sqrt{10k^2 - 25k + 25}}{k^2}$$
$$\frac{x_1 + x_2}{2} = 2 \implies \frac{\frac{2 \cdot (-5k + 2k^2 + 10)}{k^2}}{2} = 2$$
$$\frac{-5k + 2k^2 + 10}{k^2} = 2 \quad \Big/ \cdot k^2$$
$$-5k + 2k^2 + 10 = 2k^2$$
$$-5k + 10 = 0 \implies k = 2$$
$$l = 5 - 2k = 5 - 4 \implies l = 1$$
$$p \quad \ldots \quad y = 2x + 1$$