

Zadatak 41. Tjeme parabole je točka $A(-2, -1)$, a jednadžba ravnalice je $x + 2y - 1 = 0$. Odredi koordinate žarišta ove parabole.

Rješenje.

$$\text{tjeme} \dots A(-2, -1)$$

$$\text{ravnalica} \dots x + 2y - 1 = 0, \quad y = -\frac{1}{2}x + \frac{1}{2}$$

$$AF \text{ (} F \text{ fokus)} \dots k_{AF} = -\frac{1}{k_r} = -\frac{1}{-\frac{1}{2}} = 2$$

$$y - y_A = k_{AF}(x - x_A)$$

$$y + 1 = 2(x + 2)$$

$$y = 2x + 3 \implies F(x_F, 2x_F + 3)$$

$$d(A, F) = d(A, r)$$

$$\sqrt{(x_F - x_A)^2 + (y_F - y_A)^2} = \frac{|Ax_A + By_A + C|}{\sqrt{A^2 + B^2}}$$

$$\sqrt{(x_F + 2)^2 + (2x_F + 3 + 1)^2} = \frac{|1 \cdot (-2) + 2 \cdot (-1) - 1|}{\sqrt{1 + 4}}$$

$$\sqrt{(x_F + 2)^2 + (2x_F + 3 + 1)^2} = \frac{5}{\sqrt{5}} \quad /^2$$

$$(x_F + 2)^2 + (2x_F + 4)^2 = \frac{25}{5}$$

$$x_F^2 + 4x_F + 4 + 4x_F^2 + 16x_F + 16 = 5$$

$$5x_F^2 + 20x_F + 15 = 0 \quad / : 5$$

$$x_F^2 + 4x_F + 3 = 0$$

$$(x_F)_{1,2} = \frac{-4 \pm \sqrt{16 - 12}}{2} = \frac{-4 \pm 2}{2}$$

$$(x_F)_1 = -3, \quad (y_F)_1 = 2 \cdot (-3) + 3 = -3 \implies F_1(-3, -3)$$

$$(x_F)_2 = -1, \quad (y_F)_1 = 2 \cdot (-1) + 3 = 1 \implies F_2(-1, 1)$$

F_2 nije rješenje jer je bliže ravnalici od tjemena pa je rješenje: $F(-3, -3)$.