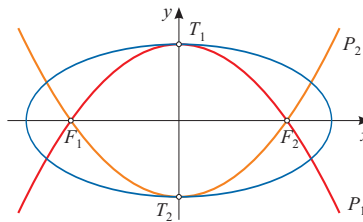


Zadatak 43. Parabola prolazi žarištima i jednim tjemenom elipse $x^2 + 5y^2 = 5$. Kako glasi njezina jednačba?

Rješenje.



$$E \dots x^2 + 5y^2 = 5 \quad / : 5$$

$$\frac{x^2}{5} + y^2 = 1$$

$$\text{žarišta elipse} \dots e^2 = a^2 - b^2 = 5 - 1 = 4 \implies e = 2, F_1(-2, 0), F_2(2, 0)$$

$$\text{tjemena elipse} \dots b^2 = 1 \implies b = \pm 1, T_1(0, 1), T_2(0, -1)$$

$$P_1 \dots T_1(0, 1)$$

$$F_1(-2, 0)$$

$$F_2(2, 0)$$

$$y - y_{T_1} = p_1(x - x_{T_1})^2 \quad (*)$$

$$F_1 \in P_1 \implies y_{F_1} - y_{T_1} = p_1(x_{F_1} - x_{T_1})^2$$

$$0 - 1 = p_1(-2 - 0)^2$$

$$-1 = 4p_1 \implies p_1 = -\frac{1}{4}$$

$$(*) \implies y - 1 = -\frac{1}{4}(x - 0)^2$$

$$y = -\frac{1}{4}x^2 + 1$$

$$P_2 \dots T_2(0, -1)$$

$$F_1(-2, 0)$$

$$F_2(2, 0)$$

$$y - y_{T_2} = p_2(x - x_{T_2})^2 \quad (**)$$

$$F_1 \in P_2 \implies y_{F_1} - y_{T_2} = p_2(x_{F_1} - x_{T_2})^2$$

$$0 + 1 = p_2(-2 - 0)^2$$

$$1 = 4p_2 \implies p_2 = \frac{1}{4}$$

$$(**) \implies y + 1 = \frac{1}{4}(x - 0)^2$$

$$y = \frac{1}{4}x^2 - 1$$