Zadatak 51. Parabola prolazi žarištima elipse $16x^2+25y^2=400$ i jednim tjemenom elipse na osi ordinata. Kako glasi jednadžba parabole?

Rješenje.

$$E \ \ldots \ 16x^2+25y^2=400 \quad / : 400$$

$$\frac{x^2}{25}+\frac{y^2}{16}=1 \implies a=5, \quad b=4, \quad e=\sqrt{25-16}=3$$

žarišta $\ldots \ F_1(-3,0), \quad F_2(3,0)$

tjemena $\ldots \ T_{1,2}(0,\pm b)$

$$T_1(0,4), \quad T_2(0,-4)$$

$P_1 \ \ldots \ T_1(0,4)$

$$F_1(-3,0)$$

$$F_2(3,0)$$

$$y-y_{T_1}=2p_1(x-x_{T_1})^2$$

$$y-4=2p_1x^2 \qquad (*)$$

$$F_1 \in P_1 \implies y-4=2p_1x^2$$

$$-4=18p_1 \implies \boxed{p_1=-\frac{2}{9}}$$

$$(*) \implies y-4=-\frac{4}{9}x^2$$

$$\boxed{x^2=-\frac{9}{4}(y-4)}$$

$P_2 \ \ldots \ T_2(0,-4)$

$$F_1(-3,0)$$

$$F_2(3,0)$$

$$y-y_{T_2}=2p_2(x-x_{T_2})^2$$

$$y+4=2p_2x^2 \qquad (**)$$

$$F_1 \in P_2 \implies y+4=2p_2x^2$$

$$4=18p_2 \implies \boxed{p_2=\frac{2}{9}}$$

$$(**) \implies y+4=\frac{4}{9}x^2$$

$$\boxed{x^2=\frac{9}{4}(y+4)}$$