Zadatak 52. Parabola $y = ax^2 + bx + c$ prolazi žarištima elipse $9x^2 + 25y^2 = 225$ i točkom $T(-2, 6)$. Kako glasi jednadžba parabole?

Rješenje.

$$P_1 \quad \ldots \quad ax^2 + bx + c = y$$

$$T(-2, 6)$$

$$E \quad \ldots \quad 9x^2 + 25y^2 = 225 \quad / : 225$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \implies a = 5, \quad b = 3, \quad e = \sqrt{25 - 9} = 4$$

$$\text{žarišta} \quad \ldots \quad F_1(-4, 0), \quad F_2(4, 0)$$

$$F_1 \in P_1 \implies a \cdot 16 + b \cdot (-4) + c = 0 \qquad (1)$$

$$F_2 \in P_1 \implies a \cdot 16 + b \cdot 4 + c = 0 \qquad (2)$$

$$T \in P_1 \implies a \cdot 4 + b \cdot (-2) + c = 6 \qquad (3)$$

$$(1) - (2) \implies -8b = 0, \quad \boxed{b = 0}$$

$$(2) - (3) \implies 12a + 6b = -6, \quad 12a = -6, \quad \boxed{a = -\frac{1}{2}}$$

$$\text{uvrstimo dobivenio u (3)} \implies -\frac{1}{2} \cdot 4 + 0 \cdot (-2) + c = 6, \quad \boxed{c = 8}$$

$$\boxed{P_1} \quad \ldots \quad \boxed{y = -\frac{1}{2}x^2 + 8}$$