Zadatak 54. Kolika je površina trokuta što ga zatvaraju asimptote hiperbole $4x^2 - 9y^2 = 36$ i ravnalica parabole $y^2 = 18x$?

Rješenje.

$$H \ \ldots \ 4x^2 - 9y^2 = 36$$
$$P \ \ldots \ y^2 = 18x$$
$$H \ \ldots \ 4x^2 - 9y^2 = 36 \quad / : 36$$
$$\frac{x^2}{9} - \frac{y^2}{4} = 1 \implies a = 3, \quad b = 2$$
$$\implies \boxed{a_1 \ \ldots \ y = \frac{2}{3}x, \quad a_2 \ \ldots \ y = -\frac{2}{3}x}$$
$$P \ \ldots \ y^2 = 18x \implies p = 9$$
$$\boxed{r \ \ldots \ x = -\frac{9}{2}}$$
$$a_1 \cap a_2 \ \ldots \ \frac{2}{3}x_A = -\frac{2}{3}x_A \implies x_A = 0, \ y_A = 0, \quad \boxed{A(0,0)}$$
$$a_1 \cap r \ \ldots \ x_B = -\frac{9}{2}, \quad y_B = \frac{2}{3} \cdot \left(-\frac{9}{2}\right) = -3 \implies \boxed{B\left(-\frac{9}{2}, -3\right)}$$
$$a_2 \cap r \ \ldots \ x_C = -\frac{9}{2}, \quad y_C = -\frac{2}{3} \cdot \left(-\frac{9}{2}\right) = 3 \implies \boxed{C\left(-\frac{9}{2}, 3\right)}$$
$$\boxed{P_\triangle} = \left|\frac{1}{2}[x_A(y_B - y_C) + x_B(y_C - y_A) + x_C(y_A - y_B)]\right|$$
$$= \frac{1}{2}\left|0 \cdot (-3 - 3) - \frac{9}{2}(3 - 0) - \frac{9}{2}(0 + 3)\right|$$
$$= \frac{1}{2}\left|-\frac{27}{2} - \frac{27}{2}\right| \quad \boxed{= \frac{27}{2}}$$