

Zadatak 60. Odredi jednadžbu kružnice koja prolazi nultočkama i tjemenom parabole $y = -x^2 + 1$.

Rješenje.

$$P \dots y = -x^2 + 1$$

$$x^2 = -y + 1$$

$$x^2 = -(y - 1) \implies T(0, 1) \text{ (tjeme)}$$

$$y = 0 \implies x^2 = 1, \quad x = \pm 1$$

$$A(-1, 0), \quad B(1, 0) \text{ (nultočke)}$$

$$k \dots (x - x_0)^2 + (y - y_0)^2 = r^2$$

$$\{T\} \in k \dots (0 - x_0)^2 + (1 - y_0)^2 = r^2$$

$$\{A\} \in k \dots (-1 - x_0)^2 + (0 - y_0)^2 = r^2$$

$$\{B\} \in k \dots (1 - x_0)^2 + (0 - y_0)^2 = r^2$$

$$x_0^2 + (1 - y_0)^2 = r^2 \quad (1)$$

$$(1 + x_0)^2 + y_0^2 = r^2 \quad (2)$$

$$(1 - x_0)^2 + y_0^2 = r^2 \quad (3)$$

$$(2) - (3) \implies (1 + x_0)^2 - (1 - x_0)^2 = 0$$

$$1 + 2x_0 + x_0^2 - 1 + 2x_0 - x_0^2 = 0$$

$$4x_0 = 0 \implies x_0 = 0$$

$$\text{uvrstimo u prve dvije jednadžbe} \implies (1 - y_0)^2 = r^2 \quad (4)$$

$$(1 + 0)^2 + y_0^2 = r^2 \quad (5)$$

$$(4) - (5) \implies (1 - y_0)^2 - 1 - y_0^2 = 0$$

$$1 - 2y_0 + y_0^2 - 1 - y_0^2 = 0$$

$$-2y_0 = 0 \implies y_0 = 0$$

$$\text{dobiveno uvrstimo u (1)} \implies 0^2 + (1 - 0)^2 = r^2$$

$$r^2 = 1$$

$$k \dots x^2 + y^2 = 1$$