

Zadatak 62. Parabola prolazi žarištima i jednim tjemenom male osi elipse $x^2 + 5y^2 = 45$. Kako glasi njezina jednadžba?

Rješenje.

$$E \dots x^2 + 5y^2 = 45 \quad / : 45$$

$$\frac{x^2}{45} + \frac{y^2}{9} = 1 \implies b = 3, \quad e = \sqrt{45 - 9} = 6$$

$$A(-6, 0), \quad B(6, 0) \in H, \quad T_1(0, 3), \quad T_2(0, -3) \text{ tjemena}$$

Imamo dva slučaja:

1) Parabola određena s T_1, A, B

2) Parabola određena s T_2, A, B

1) $P_1 \dots T_1(0, 3)$ (tjeme)

$$A(-6, 0)$$

$$B(6, 0)$$

$$(x - x_0)^2 = 2p(y - y_0)$$

$$x^2 = 2p(y - 3)$$

$$\{A\} \in P \implies 36 = 2p(0 - 3)$$

$$36 = -6p \implies p = -6$$

$$x^2 = -12(y - 3) \implies y = -\frac{1}{12}x^2 + 3$$

2) $P_2 \dots T_2(0, -3)$ (tjeme)

$$A(-6, 0)$$

$$B(6, 0)$$

$$(x - x_0)^2 = 2p(y - y_0)$$

$$x^2 = 2p(y + 3)$$

$$\{A\} \in P \implies 36 = 2p(0 + 3)$$

$$36 = 6p \implies p = 6$$

$$x^2 = 12(y + 3) \implies y = \frac{1}{12}x^2 - 3$$