Zadatak 7. Prikaži grafički funkcije:

1) $f(x) = \frac{1}{2}\operatorname{tg}\left(\frac{x}{2} - \pi\right)$;

2) $f(x) = -\operatorname{ctg}\left(2x - \frac{\pi}{4}\right)$;

3) $f(x) = \left|\operatorname{tg}\left(2x + \frac{3\pi}{2}\right)\right|$;

4) $f(x) = -|\operatorname{ctg}|x + \pi||$.

Rješenje. **1)** $f(x) = \frac{1}{2}\operatorname{tg}\left(\frac{x}{2} - \pi\right) = \frac{1}{2}\operatorname{tg}\frac{1}{2}\left(x - \frac{\pi}{2}\right)$;

vertikalne asimptote:

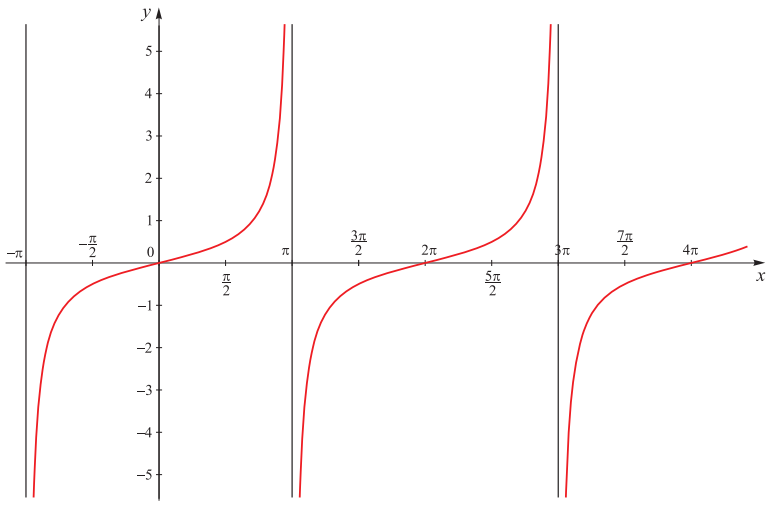
$$\operatorname{tg}\frac{1}{2}\left(x - \frac{\pi}{2}\right) = \pm\infty$$
$$\frac{1}{2}\left(x - \frac{\pi}{2}\right) = \frac{\pi}{2} + k\pi \quad / \cdot 2$$
$$x - \frac{\pi}{2} = \pi + 2k\pi$$
$$x = \frac{3\pi}{2} + 2k\pi$$

nultočke:

$$\operatorname{tg}\frac{1}{2}\left(x - \frac{\pi}{2}\right) = 0$$
$$\frac{1}{2}\left(x - \frac{\pi}{2}\right) = k\pi \quad / \cdot 2$$
$$x - \frac{\pi}{2} = 2k\pi$$
$$x = \frac{\pi}{2} + 2k\pi$$

period:

$$P = \frac{\pi}{\frac{1}{2}} = 2\pi$$



2) $f(x) = -\operatorname{ctg}\left(2x - \frac{\pi}{4}\right) = -\operatorname{ctg}2\left(x - \frac{\pi}{8}\right)$;

vertikalne asimptote:

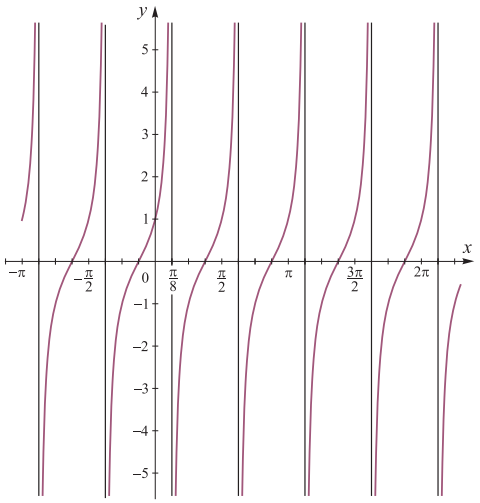
$$\operatorname{ctg}2\left(x - \frac{\pi}{8}\right) = \pm\infty$$
$$2\left(x - \frac{\pi}{8}\right) = k\pi \quad / : 2$$
$$x - \frac{\pi}{8} = \frac{k\pi}{2}$$
$$x = \frac{\pi}{8} + \frac{k\pi}{2}$$

nultočke:

$$\operatorname{ctg}2\left(x - \frac{\pi}{8}\right) = 0$$
$$2\left(x - \frac{\pi}{8}\right) = \frac{\pi}{2} + k\pi \quad / : 2$$
$$x - \frac{\pi}{8} = \frac{\pi}{4} + \frac{k\pi}{2}$$
$$x = \frac{3\pi}{8} + \frac{k\pi}{2}$$

period:

$$P = \frac{\pi}{2}$$



3) $f(x) = \left|\operatorname{tg}\left(2x+\frac{3\pi}{2}\right)\right| = \frac{1}{\left|\operatorname{ctg}\left(2x+\frac{3\pi}{2}\right)\right|} = \frac{1}{\left|\frac{\operatorname{ctg} 2x \operatorname{ctg}\frac{3\pi}{2} - 1}{\operatorname{ctg}\frac{3\pi}{2} + \operatorname{ctg} 2x}\right|} =$

$\frac{1}{\left|\frac{-1}{\operatorname{ctg} 2x}\right|} = |\operatorname{ctg} 2x|;$

vertikalne asimptote:

$\operatorname{ctg} 2x = \pm\infty$

$2x = k\pi \quad / : 2$

$x = \frac{k\pi}{2}$

nultočke:

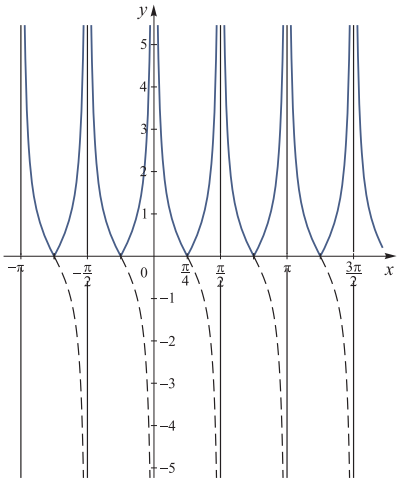
$\operatorname{ctg} 2x = 0$

$2x = \frac{\pi}{2} + k\pi \quad / : 2$

$x = \frac{\pi}{4} + \frac{k\pi}{2}$

period:

$P = \pi$



4)

$$f(x) = -|\operatorname{ctg}|x+\pi|| = \begin{cases} -|-\operatorname{ctg}(x-\pi)| & x < -\pi \\ -|\operatorname{ctg}(x+\pi)| & x \geqslant -\pi \end{cases} = \begin{cases} -|\operatorname{ctg}(x-\pi)| & x < -\pi \\ -|\operatorname{ctg}(x+\pi)| & x \geqslant -\pi \end{cases}$$

$x < -\pi$:

vertikalne asimptote:

$$x - \pi = k\pi$$
$$x = \pi + k\pi$$

nultočke:

$$x - \pi = \frac{\pi}{2} + k\pi$$
$$x = \frac{3\pi}{2} + k\pi$$

period:

$$P = \pi$$

$x \geqslant -\pi$:

vertikalne asimptote:

$$x + \pi = k\pi$$
$$x = -\pi + k\pi$$

nultočke:

$$x + \pi = \frac{\pi}{2} + k\pi$$
$$x = \frac{\pi}{2} + k\pi$$

period:

$$P = \pi$$