Rješenja složenijih zadataka

Zadatak 1. Prikaži grafički sljedeće funkcije:

1) $f(x) = \sin x \cdot \sqrt{\cos^2 x} + \cos x \cdot \sqrt{\sin^2 x}$;

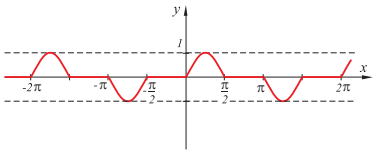
2) $f(x) = \sin 2x + \sqrt{3}\cos 2x$;

3) $f(x) = |\sin 2x| + \sqrt{3}\cos 2x$;

4) $f(x) = |\sin 2x| + \sqrt{3}|\cos 2x|$.

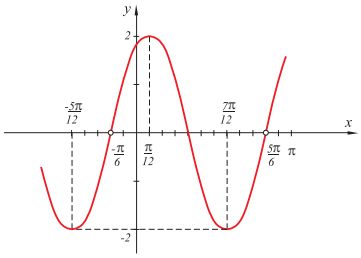
Rješenje. **1)** $f(x) = \sin x \cdot \sqrt{\cos^2 x} + \cos x \cdot \sqrt{\sin^2 x} = \sin x \cdot |\cos x| + \cos x \cdot |\sin x|$

$$f(x) = \begin{cases} \sin x \cdot \cos x + \cos x \cdot \sin x & 0 \leqslant x < \dfrac{\pi}{2} \\ -\sin x \cdot \cos x + \cos x \cdot \sin x & \dfrac{\pi}{2} \leqslant x < \pi \\ -\sin x \cdot \cos x - \cos x \cdot \sin x & \pi \leqslant x < \dfrac{3\pi}{2} \\ \sin x \cdot \cos x - \cos x \cdot \sin x & \dfrac{3\pi}{2} \leqslant x < 2\pi \end{cases} = \begin{cases} \sin 2x & x \text{ u I. kvadrantu} \\ 0 & x \text{ u II. kvadrantu} \\ -\sin 2x & x \text{ u III. kvadrantu} \\ 0 & x \text{ u IV. kvadrantu} \end{cases}$$



2) $f(x) = \sin 2x + \sqrt{3}\cos 2x = 2\left(\frac{1}{2}\sin 2x + \cos 2x \cdot \frac{\sqrt{3}}{2}\cos 2x\right) = 2\left(\sin 2x \cos\frac{\pi}{3} + \cos 2x \sin\frac{\pi}{3}\right) = 2\sin\left(2x + \frac{\pi}{3}\right)$

(slika).



3) $f(x) = |\sin 2x| + \sqrt{3}\cos 2x$

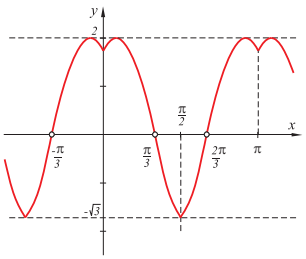
$$\sin 2x \geqslant 0 \;\; \forall 2x \in [2k\pi, (2k+1)\pi], \; x \in \left[k\pi, \frac{(2k+1)}{2}\pi\right]$$

$$f(x) = \begin{cases} \sin 2x + \sqrt{3}\cos 2x & x \in \left[k\pi, \frac{(2k+1)}{2}\pi\right] \\ -\sin 2x + \sqrt{3}\cos 2x & x \notin \left[k\pi, \frac{(2k+1)}{2}\pi\right] \end{cases}$$

$$= \begin{cases} 2\left(\frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x\right) & x \in \left[k\pi, \frac{(2k+1)}{2}\pi\right] \\ 2\left(-\frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x\right) & x \notin \left[k\pi, \frac{(2k+1)}{2}\pi\right] \end{cases}$$

$$= \begin{cases} 2\left(\sin 2x \cos\frac{\pi}{3} + \cos 2x \sin\frac{\pi}{3}\right) & x \in \left[k\pi, \frac{(2k+1)}{2}\pi\right] \\ 2\left(-\sin 2x \cos\frac{\pi}{3} + \cos 2x \cos\frac{\pi}{3}\right) & x \notin \left[k\pi, \frac{(2k+1)}{2}\pi\right] \end{cases}$$

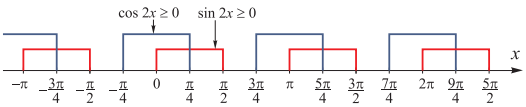
$$= \begin{cases} 2\sin\left(2x + \frac{\pi}{3}\right) & x \in \left[k\pi, \frac{(2k+1)}{2}\pi\right] \\ -2\sin\left(2x - \frac{\pi}{3}\right) & x \notin \left[k\pi, \frac{(2k+1)}{2}\pi\right] \end{cases}$$



4) $f(x) = |\sin 2x| + \sqrt{3}|\cos 2x|$

$$\sin 2x \geqslant 0 \;\; \forall 2x \in [2k\pi, (2k+1)\pi], \; x \in \left[k\pi, \frac{(2k+1)}{2}\pi\right]$$

$$\cos 2x \geqslant 0 \;\; \forall 2x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right], \; x \in \left[-\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi\right]$$



$$\sin 2x \geqslant 0, \; \cos 2x \geqslant 0 \;\; x \in \left[k\pi, \frac{4k+1}{4}\pi\right]$$

$$\sin 2x < 0, \; \cos 2x > 0 \;\; x \in \left[\frac{4k-1}{4}\pi, k\pi\right]$$

$$\sin 2x > 0, \; \cos 2x < 0 \;\; x \in \left[\frac{4k+1}{4}\pi, \frac{2k+1}{2}\pi\right]$$

$$\sin 2x \leqslant 0, \; \cos 2x \leqslant 0 \;\; x \in \left[\frac{2k+1}{2}\pi, \frac{4k+3}{4}\pi\right]$$

$$f(x)=\begin{cases} 2\left(\frac{1}{2}\sin 2x+\frac{\sqrt{3}}{2}\cos 2x\right) & x\in\left[k\pi,\frac{4k+1}{4}\pi\right] \\ 2\left(-\frac{1}{2}\sin 2x+\frac{\sqrt{3}}{2}\cos 2x\right) & x\in\left[\frac{4k-1}{4}\pi,k\pi\right] \\ 2\left(\frac{1}{2}\sin 2x-\frac{\sqrt{3}}{2}\cos 2x\right) & x\in\left[\frac{4k+1}{4}\pi,\frac{2k+1}{2}\pi\right] \\ 2\left(-\frac{1}{2}\sin 2x-\frac{\sqrt{3}}{2}\cos 2x\right) & x\in\left[\frac{2k+1}{2}\pi,\frac{4k+3}{4}\pi\right] \end{cases}$$

$$=\begin{cases} 2\sin\left(2x+\frac{\pi}{3}\right) & x\in\left[k\pi,\frac{4k+1}{4}\pi\right] \\ -2\sin\left(2x-\frac{\pi}{3}\right) & x\in\left[\frac{4k-1}{4}\pi,k\pi\right] \\ 2\sin\left(2x+\frac{\pi}{3}\right) & x\in\left[\frac{4k+1}{4}\pi,\frac{2k+1}{2}\pi\right] \\ -2\sin\left(2x+\frac{\pi}{3}\right) & x\in\left[\frac{2k+1}{2}\pi,\frac{4k+3}{4}\pi\right] \end{cases}$$