Zadatak 5. Koliko rješenja ima jednadžba:

1) $|\cos(\pi x)| = |1 - |x||$;

2) $\sin\frac{\pi}{2}(1-x) = |x^2 - 3x|$;

3) $2\left|\sin\frac{\pi x}{2}\right| = |\log_{\frac{1}{2}} x|$;

4) $2\sin\left|\frac{\pi x}{2}\right| = \left|\log_{\frac{1}{2}}|x|\right|$?

Rješenje. **1)** $|\cos(\pi x)| = |1 - |x||$

$$f(x) = |\cos(\pi x)| = \begin{cases} \cos\pi x & -\frac{\pi}{2} + 2k\pi \leqslant \pi x \leqslant \frac{\pi}{2} + 2k\pi \\ -\cos\pi x & \frac{\pi}{2} + 2k\pi \leqslant \pi x \leqslant \frac{3\pi}{2} + 2k\pi \end{cases}$$

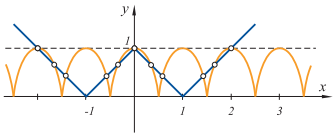
$$= \begin{cases} \cos\pi x & -\frac{1}{2} + 2k \leqslant x \leqslant \frac{1}{2} + 2k \\ -\cos\pi x & \frac{1}{2} + 2k \leqslant x \leqslant \frac{3}{2} + 2k \end{cases} \quad \text{period}$$

$$P = \frac{2\pi}{\pi} = 2$$

$$g(x) = |1 - |x|| = \begin{cases} 1 - |x| & 1 - |x| \geqslant 0 \\ -1 + |x| & 1 - |x| < 0 \end{cases} = \begin{cases} 1 - |x| & -1 \leqslant x \leqslant 1 \\ -1 + |x| & x < -1,\ x > 1 \end{cases}$$

$$= \begin{cases} 1 - x & 0 < x \leqslant 1 \\ 1 + x & -1 \leqslant x \leqslant 0 \\ -1 + x & x > 1 \\ -1 - x & x < -1 \end{cases}$$

11 rješenja (vidi sliku!)

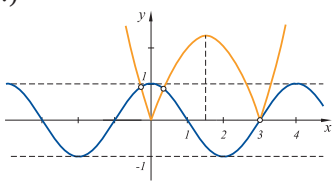


2) $\sin\frac{\pi}{2}(1-x) = |x^2 - 3x|$, $\quad -\sin\frac{\pi}{2}(x-1) = |x^2 - 3x|$;

$f(x) = -\sin\frac{\pi}{2}(x-1)$, period $P = \frac{2\pi}{\frac{\pi}{2}} = 4$;

$g(x) = |x^2 - 3x|$, nultočke $x^2 - 3x = 0$, $x(x-3) = 0$, $x_1 = 0$, $x_2 = 3$

3 rješenja (vidi sliku!)

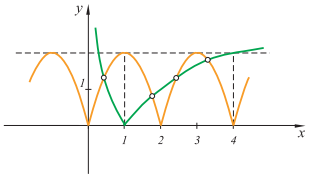


3) $2\left|\sin\frac{\pi x}{2}\right| = |\log_{\frac{1}{2}} x|$;

$f(x) = 2\left|\sin\frac{\pi x}{2}\right|$, amplituda je 2, $0 \leqslant f(x) \leqslant 2$, period $P = \frac{2\pi}{\frac{\pi}{2}} = 4$;

$g(x) = |\log_{\frac{1}{2}} x|$;

4 rješenja (vidi sliku!)



4) $2\sin\left|\frac{\pi x}{2}\right| = \left|\log_{\frac{1}{2}}|x|\right|$;

$f(x) = 2\sin\left|\frac{\pi x}{2}\right|$, amplituda je 2, $-2 \leqslant f(x) \leqslant 2$, period $P = \frac{2\pi}{\frac{\pi}{2}} = 4$;

$g(x) = |\log_{\frac{1}{2}}|x||$;

4 rješenja (vidi sliku!)