Zadatak 6. Prikaži grafički skup točaka ravnine za čije koordinate x i y vrijedi:

1) $f(x) = \cos^2(\sqrt{\operatorname{tg} x}) + \sin^2(\sqrt{\operatorname{tg} x})$;

2) $f(x) = \operatorname{tg} x \cdot \operatorname{ctg} x$;

3) $f(x) = \dfrac{\sqrt{1-\sin^2 x}}{\cos x}$;

4) $f(x) = \dfrac{\sqrt{1-\cos^2 x}}{\cos x}$;

5) $f(x) = \cos|x| - |\cos x|$;

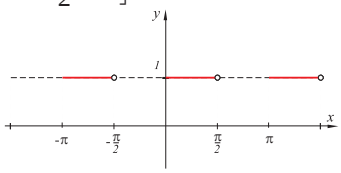
6) $f(x) = \sin x \cdot \sqrt{\dfrac{1}{\sin^2 x} - 1}$;

7) $f(x) = |\cos x| \cdot \operatorname{tg}|x|$;

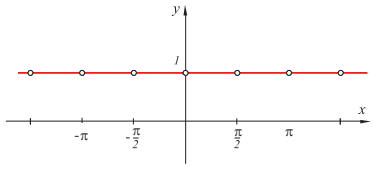
8) $f(x) = |\sin x| \cdot \operatorname{ctg}|x|$.

Rješenje. **1)** $f(x) = \cos^2(\sqrt{\operatorname{tg} x}) + \sin^2(\sqrt{\operatorname{tg} x})$;

$f(x) = 1$ za $x \in \left[k\pi, \dfrac{2k+1}{2}\pi\right]$ gdje je $\operatorname{tg} x \geqslant 0$

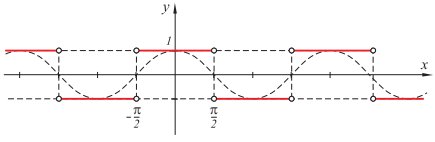


2) $f(x) = \operatorname{tg} x \cdot \operatorname{ctg} x = 1$ za $x \in \mathbf{R} \setminus \left\{k\pi, \dfrac{k\pi}{2};\ k \in \mathbf{Z}\right\}$ gdje $\operatorname{tg} x$ odnosno $\operatorname{ctg} x$ nisu definirani.;



3) $f(x) = \dfrac{\sqrt{1-\sin^2 x}}{\cos x} = \dfrac{\pm\cos x}{\cos x} = \pm 1$;

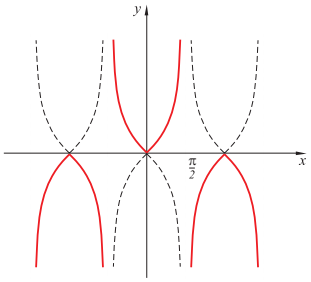
$$f(x) = \begin{cases} 1, & \cos x \geqslant 0 \\ -1, & \cos x < 0 \end{cases} = \begin{cases} 1, & -\dfrac{\pi}{2} + 2k\pi \leqslant x \leqslant \dfrac{\pi}{2} + 2k\pi \\ -1, & \dfrac{\pi}{2} + 2k\pi < x < \dfrac{3\pi}{2} + 2k\pi \end{cases}$$



4) $f(x) = \dfrac{\sqrt{1-\cos^2 x}}{\cos x} = \dfrac{\pm\sin x}{\cos x} = \pm\operatorname{tg} x$

$$f(x) = \begin{cases} \operatorname{tg} x, & \sin x \geqslant 0 \\ -\operatorname{tg} x, & \sin x < 0 \end{cases} = \begin{cases} \operatorname{tg} x, & 2k\pi \leqslant x \leqslant (2k+1)\pi \\ -\operatorname{tg} x, & (2k-1)\pi < x < 2k\pi \end{cases}$$

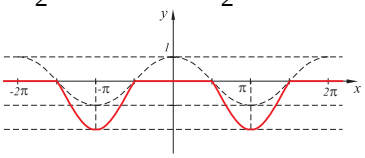
$x \neq \frac{k\pi}{2}$ jer mora vrijediti $\cos x \neq 0$



5) $f(x) = \cos|x| - |\cos x| = \cos x - |\cos x|$;

$$f(x) = \begin{cases} \cos x - \cos x, & -\frac{\pi}{2} + 2k\pi \leqslant x \leqslant \frac{\pi}{2} + 2k\pi \\ \cos x + \cos x, & \frac{\pi}{2} + 2k\pi < x < \frac{3\pi}{2} + 2k\pi \end{cases}$$

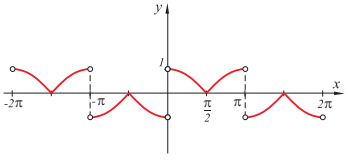
$$= \begin{cases} 0, & -\frac{\pi}{2} + 2k\pi \leqslant x \leqslant \frac{\pi}{2} + 2k\pi \\ 2\cos xx, & \frac{\pi}{2} + 2k\pi < x < \frac{3\pi}{2} + 2k\pi \end{cases}$$



6) $f(x) = \sin x \cdot \sqrt{\frac{1}{\sin^2 x} - 1} = \sin x \cdot \sqrt{\frac{1 - \sin^2 x}{\sin^2 x}} = \sin x \cdot \sqrt{\frac{\cos^2 x}{\sin^2 x}} =$
$\sin x \cdot (\pm \operatorname{ctg} x) = \pm \sin x \cdot \frac{\cos x}{\sin x} = \pm \cos x$;

$$f(x) = \begin{cases} \cos x, & \operatorname{ctg} x \geqslant 0 \\ -\cos x, & \operatorname{ctg} x < 0 \end{cases} = \begin{cases} \cos x, & k\pi \leqslant x \leqslant \frac{2k+1}{2}\pi \\ -\cos x, & \frac{2k-1}{2}\pi < x < k\pi \end{cases}$$

$x \neq k\pi$ jer mora vrijediti $\sin x \neq 0$



7) $f(x) = |\cos x| \cdot \operatorname{tg}|x|$;

$$g(x) = |\cos x| = \begin{cases} \cos x, & x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right] \\ -\cos x, & x \in \left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\rangle \end{cases}$$

$$h(x) = \operatorname{tg}|x| = \begin{cases} \operatorname{tg} x, & x \geqslant 0 \\ -\operatorname{tg} x, & x < 0 \end{cases}$$

$\underline{1^\circ \quad x \geqslant 0:}$

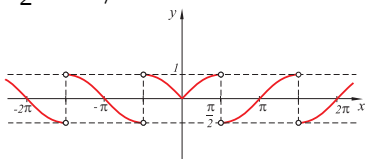
$x \in \left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2} + 2k\pi, \frac{5\pi}{2} + 2k\pi\right] \ (k \geqslant 0) \implies f(x) = \cos x \cdot \operatorname{tg} x = \sin x;$

$x \in \left\langle \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2\kappa\pi \right\rangle \ (k \geqslant 0) \implies f(x) = -\cos x \cdot \operatorname{tg} x = -\sin x;$

$\underline{2^\circ \quad x < 0:}$

$x \in \left[-\frac{\pi}{2}, 0\right\rangle \cup \left[-\frac{5\pi}{2} - 2k\pi, -\frac{3\pi}{2} - 2k\pi,\right] \ (k \geqslant 0) \implies f(x) = \cos x \cdot (-\operatorname{tg} x) = -\sin x;$

$x \in \left\langle -\frac{3\pi}{2} - 2k\pi, -\frac{\pi}{2} - 2k\pi \right\rangle \ (k \geqslant 0) \implies f(x) = -\cos x \cdot (-\operatorname{tg} x) = \sin x;$



8) $f(x) = |\sin x| \cdot \operatorname{ctg} |x|;$

$$g(x) = |\sin x| = \begin{cases} \sin x, & x \in [0 + 2k\pi, \pi + 2k\pi] \\ -\sin x, & x \in \langle \pi + 2k\pi, 2\pi + 2k\pi \rangle \end{cases}$$

$$h(x) = \operatorname{ctg} |x| = \begin{cases} \operatorname{ctg} x, & x \geqslant 0 \\ -\operatorname{ctg} x, & x < 0 \end{cases}$$

$\underline{1^\circ \quad x \geqslant 0:}$

$x \in [0 + 2k\pi, \pi + 2k\pi] \ (k \geqslant 0) \implies f(x) = \sin x \cdot \operatorname{ctg} x = \cos x;$

$x \in \langle \pi + 2k\pi, 2\pi + 2k\pi \rangle \ (k \geqslant 0) \implies f(x) = -\sin x \cdot \operatorname{ctg} x = -\cos x;$

$\underline{2^\circ \quad x < 0:}$

$x \in [-2\pi - 2k\pi, -\pi - 2k\pi] \ (k \geqslant 0) \implies f(x) = \sin x \cdot (-\operatorname{ctg} x) = -\cos x;$

$x \in \langle -\pi - 2k\pi, -2k\pi \rangle \ (k \geqslant 0) \implies f(x) = -\sin x \cdot (-\operatorname{ctg} x) = \cos x;$