Zadatak 7. Odredi skup svih točaka ravnine čije koordinate x i y zadovoljavaju sljedeći uvjet:

1) $\sin^2(\pi x) + 1 = \cos^2(\pi y)$;

2) $|y| = \dfrac{\cos x}{|\cos x|}$;

3) $|x| = x \cdot \cos(2\pi y)$;

4) $\cos(\pi(|x| + |y|)) = 0$.

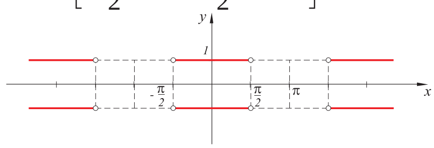
Rješenje. **1)** Iz dane jednakosti slijedi $\sin^2(\pi x) = 0$ i $\cos^2(\pi y) = 1$, a odatle $\pi x = k\pi$ i $\pi y = n\pi$, k, $n \in \mathbf{Z}$. Graf čine sve točke ravnine s cjelobrojnim koordinatama.

2)

$$|y| = \frac{\cos x}{|\cos x|} = \begin{cases} \dfrac{\cos x}{\cos x}, & x \in \left[-\dfrac{\pi}{2} + 2k\pi, \dfrac{\pi}{2} + 2k\pi\right] \\ -\dfrac{\cos x}{\cos x}, & x \in \left\langle \dfrac{\pi}{2} + 2k\pi, \dfrac{3\pi}{2} + 2k\pi \right\rangle \end{cases} = \begin{cases} 1, & x \in \left[-\dfrac{\pi}{2} + 2k\pi, \dfrac{\pi}{2} + 2k\pi\right] \\ -1, & x \in \left\langle \dfrac{\pi}{2} + 2k\pi, \dfrac{3\pi}{2} + 2k\pi \right\rangle \end{cases}$$

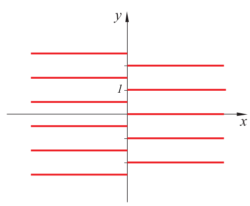
$|y| = -1$ nije rješenje jer $|y| \geqslant 0$, $\forall y \in \mathbf{R}$.

$$\Longrightarrow y = \pm 1 \text{ za } x \in \left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right].$$



3) $|x| = x \cdot \cos(2\pi y)$

$$\frac{|x|}{x} = \cos(2\pi y) \Longrightarrow \cos(2\pi y) = \begin{cases} 1, & x \geqslant 0 \\ -1, & x < 0 \end{cases} \Longrightarrow y = \begin{cases} k, & \geqslant 0 \\ \dfrac{2k+1}{2}, & x < 0 \end{cases}$$



4)

$$\cos(\pi(|x| + |y|)) = 0$$

$$\pi(|x| + |y|) = \frac{(2k+1)\pi}{2} \quad / : \pi$$

$$|x| + |y| = \frac{2k+1}{2}$$

$$|y| = \frac{2k+1}{2} - |x| = \begin{cases} -x + \dfrac{2k+1}{2}, & x < 0 \\ x + \dfrac{2k+1}{2}, & x \geqslant 0 \end{cases}$$

$$|y| \geqslant 0 \implies \text{funkcija je definirana za } -x+\frac{2k+1}{2} \geqslant 0 \text{ i } x+\frac{2k+1}{2} \geqslant 0\,.$$

$$-x+\frac{2k+1}{2} \geqslant 0 \implies x \leqslant \frac{2k+1}{2} \text{ i } x<0,\ \forall k \in \mathbf{Z}$$

$$x+\frac{2k+1}{2} \geqslant 0 \implies x \geqslant -\frac{2k+1}{2} \text{ i } x \geqslant 0,\ \ x \geqslant -k-\frac{1}{2} \text{ i } x \geqslant 0,\ \ \forall k \geqslant 1$$

$$y=\begin{cases} \pm\left(-x+\dfrac{2k+1}{2}\right), & x<0,\ \ \forall k \geqslant 0 \\ \pm\left(x+\dfrac{2k+1}{2}\right), & x \geqslant 0,\ \ \forall k \geqslant 1 \end{cases}$$