Zadatak 24. Trokut ABC je pravokutan. Provjeri! Kako glasi jednadžba kružnice opisane tom trokutu ako je:

1) $A(-2, -1)$, $B(4, 1)$, $C(0, 3)$;

2) $A(-5, -6)$, $B(-5, 7)$, $C(1, 3)$;

3) $A(-3, 1)$, $B(-4, 4)$, $C(3, 3)$.

Rješenje. **1)** $k_{AC} = \dfrac{3+1}{0+2} = 2$, $k_{BC} = \dfrac{3-1}{0-4} = -\dfrac{1}{2} \implies \overline{AC} \perp \overline{BC}$.

Središte pravokutnom trokutu opisane kružnice je u polovištu hipotenuze.

$$S\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = S\left(\frac{-2+4}{2}, \frac{-1+1}{2}\right) = S(1, 0).$$

$$r = \frac{d(A, B)}{2} = \frac{\sqrt{(-2-4)^2 + (-1-1)^2}}{2} = \frac{\sqrt{6^2 + 2^2}}{2} = \frac{\sqrt{40}}{2} = \sqrt{10}.$$

$$(x-1)^2 + y^2 = 10.$$

2) $k_{AC} = \dfrac{3-7}{1+5} = -\dfrac{2}{3}$, $k_{BC} = \dfrac{3+6}{1+5} = \dfrac{3}{2} \implies \overline{AC} \perp \overline{BC}$.

$$S\left(\frac{-5-5}{2}, \frac{-6+7}{2}\right) = S\left(-5, \frac{1}{2}\right).$$

$$r = \frac{d(A, B)}{2} = \frac{\sqrt{(-5+5)^2 + (7+6)^2}}{2} = \frac{13}{2}.$$

$$(x+5)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{169}{4}.$$

3) $k_{AC} = \dfrac{3-1}{3+3} = \dfrac{1}{3}$, $k_{AB} = \dfrac{4-1}{-4+3} = -3 \implies \overline{AC} \perp \overline{AB}$.

$$S\left(\frac{-4+3}{2}, \frac{4+3}{2}\right) = S\left(-\frac{1}{2}, \frac{7}{2}\right).$$

$$r = \frac{d(B, C)}{2} = \frac{\sqrt{(3+4)^2 + (3-4)^2}}{2} = \frac{\sqrt{50}}{2} = \frac{5\sqrt{2}}{2}.$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{50}{4}$$

$$\left(\frac{2x+1}{2}\right)^2 + \left(\frac{2y-7}{2}\right)^2 = \frac{50}{4}$$

$$(2x+1)^2 + (2y-7)^2 = 50$$