Zadatak 45. Kako glasi jednadžba kružnice opisane trokutu ABC ako je:

1) $A(-4,4)$, $B(1,-1)$, $C(1,5)$;
2) $A(-3,2)$, $B(4,1)$, $C(5,-2)$;
3) $A(-8,3)$, $B(1,0)$, $C(-1,4)$;
4) $A(1,2)$, $B(3,-4)$, $C(6,2)$;
5) $A(2,6)$, $B(9,7)$, $C(3,-1)$;
6) $A(-3,-5)$, $B(4,-6)$, $C(5,1)$?

Rješenje. **1)**

$$\begin{aligned}
(-4-p)^2+(4-q)^2&=r^2\\
(1-p)^2+(-1-q)^2&=r^2\\
(1-p)^2+(5-q)^2&=r^2
\end{aligned}$$

$$\begin{aligned}
16+8p+p^2+16-8q+q^2&=r^2\\
1-2p+p^2+1+2q+q^2&=r^2\\
1-2p+p^2+25-10q+q^2&=r^2
\end{aligned}$$

$$\begin{aligned}
p^2+q^2+8p-8q+32&=r^2\\
p^2+q^2-2p+2q+2&=r^2\\
p^2+q^2-2p-10q+26&=r^2
\end{aligned}$$

$$\begin{aligned}
10p-10q+30&=0\\
12q-24&=0
\end{aligned}$$

$$\begin{aligned}
p-q&=-3\\
q&=2\\
p&=-1\\
(-1)^2+2^2-2(-1)-10\cdot 2+26&=r^2\\
1+4+2-20+26&=r^2\\
r^2&=13
\end{aligned}$$

$(x+1)^2+(y-2)^2=13$;

2)

$$\begin{aligned}(-3-p)^2+(2-q)^2&=r^2\\(4-p)^2+(1-q)^2&=r^2\\(5-p)^2+(-2-q)^2&=r^2\end{aligned}$$

$$\begin{aligned}9+6p+p^2+4-4q+q^2&=r^2\\16-8p+p^2+1-2q+q^2&=r^2\\25-10p+p^2+4+4q+q^2&=r^2\end{aligned}$$

$$\begin{aligned}p^2+q^2+6p-4q+13&=r^2\\p^2+q^2-8p-2q+17&=r^2\\p^2+q^2-10p+4q+29&=r^2\end{aligned}$$

$$\begin{aligned}14p-2q-4&=0\\2p-6q-12&=0\end{aligned}$$

$$\begin{aligned}7p-q&=2 \implies q=7p-2\\p-3q&=6\end{aligned}$$

$$\begin{aligned}p-21p+6&=6\\p&=0\\q&=-2\\0^2+(-2)^2+6\cdot 0-4\cdot(-2)+13&=r^2\\0+4+0+8+13&=r^2\\r^2&=25\end{aligned}$$

$x^2+(y+2)^2=25\,;$

3)

$$\begin{aligned}
(-8-p)^2+(3-q)^2 &= r^2\\
(1-p)^2+(0-q)^2 &= r^2\\
(-1-p)^2+(4-q)^2 &= r^2
\end{aligned}$$

$$\begin{aligned}
64+16p+p^2+9-6q+q^2 &= r^2\\
1-2p+p^2+q^2 &= r^2\\
1+2p+p^2+16-8q+q^2 &= r^2
\end{aligned}$$

$$\begin{aligned}
p^2+q^2+16p-6q+73 &= r^2\\
p^2+q^2-2p+1 &= r^2\\
p^2+q^2+2p-8q+17 &= r^2
\end{aligned}$$

$$\begin{aligned}
18p-6q+72 &= 0\\
-4p+8q-16 &= 0
\end{aligned}$$

$$\begin{aligned}
9p-3q &= -36\\
p-2q &= -4 \implies p = 2q-4
\end{aligned}$$

$$\begin{aligned}
18q-36+3q &= 36\\
q &= 0\\
p &= -4\\
(-4)^2+0^2-2\cdot(-4)+1 &= r^2\\
16+8+1 &= r^2\\
r^2 &= 25
\end{aligned}$$

$(x+4)^2+y^2=25$;

4)

$$(1-p)^2+(2-q)^2=r^2$$
$$(3-p)^2+(-4-q)^2=r^2$$
$$(6-p)^2+(2-q)^2=r^2$$

$$1-2p+p^2+4-4q+q^2=r^2$$
$$9-6p+p^2+16+8q+q^2=r^2$$
$$36-12p+p^2+4-4q+q^2=r^2$$

$$p^2+q^2-2p-4q+5=r^2$$
$$p^2+q^2-6p+8q+25=r^2$$
$$p^2+q^2-12p-4q+40=r^2$$

$$4p-12q-20=0$$
$$6p+12q-15=0$$

$$10p-35=0$$
$$p=\frac{7}{2}$$
$$12q=15-6\cdot\frac{7}{2}$$
$$q=\frac{-6}{12}$$
$$q=-\frac{1}{2}$$
$$\left(\frac{7}{2}\right)^2+\left(-\frac{1}{2}\right)^2-2\cdot\frac{7}{2}-4\cdot\left(-\frac{1}{2}\right)+5=r^2$$
$$\frac{1}{4}+\frac{49}{4}-7+2+5=r^2$$
$$r^2=\frac{25}{2}$$

$x^2+y^2-7x+y=0\,;$

5) $x^2 + y^2 + ax + by + c = 0$

$$\begin{aligned}4 + 36 + 2a + 6b + c &= 0\\ 81 + 49 + 9a + 7b + c &= 0\\ 9 + 1 + 3a - b + c &= 0\end{aligned}$$

$$\begin{aligned}2a + 6b + c &= -40\\ 9a + 7b + c &= -130\\ 3a - b + c &= -10 \implies b = 3a + c + 10\end{aligned}$$

$$\begin{aligned}2a + 18a + 6c + 60 + c &= -40\\ 9a + 21a + 7c + 70 + c &= -130\end{aligned}$$

$$\begin{aligned}20a + 7c &= -100 \cdot (-3)\\ 30a + 8c &= -200 \cdot 2\end{aligned}$$

$$\begin{aligned}-60a - 21c &= 300\\ 60a + 16c &= -400\end{aligned}$$

$$\begin{aligned}-5c &= -100\\ c &= 20\\ 20a + 7 \cdot 20 &= -100\\ a &= -12\\ b &= -36 + 20 + 10 = -6\end{aligned}$$

$x^2 + y^2 - 12x - 6y + 20 = 0$;

6)

$$9+25-3a-5b+c=0$$
$$16+36+4a-6b+c=0$$
$$25+1+5a+b+c=0$$

$$-3a-5b+c=-34$$
$$4a-6b+c=-52$$
$$5a+b+c=-26$$

$$-7a+b=18 \implies b=18+7a$$
$$-a-7b=-26$$

$$-a-126-49a=-26$$
$$-50a=100$$
$$a=-2$$
$$b=18-14=4$$
$$6-20+c=-34$$
$$c=-20$$

$x^2+y^2-2x+4y-20=0$.