

Zadatak 52. Koliki kut zatvaraju tangente povučene iz ishodišta koordinatnog sustava na kružnicu $x^2 + y^2 - 4x - 12y + 36 = 0$?

Rješenje.

$$\begin{aligned} -4 &= -2p \implies p = 2 \\ -12 &= -2q \implies q = 6, S(2, 6) \\ 36 &= 4 + 36 - r^2 \implies r^2 = 4 \end{aligned}$$

$$y = kx$$

$$x^2 + k^2x^2 - 4x - 12kx + 36 = 0$$

$$x^2(1 + k^2) - x(4 + 12k) + 36 = 0$$

$$x_{1,2} = \frac{-4 - 12k \pm \sqrt{(4 + 12k)^2 - 4 \cdot 36 \cdot (1 + k^2)}}{2}$$

$$\implies (4 + 12k)^2 - 144(1 + k^2) = 0$$

$$16 + 96k + 144k^2 - 144 - 144k^2 = 0$$

$$96k - 128 = 0$$

$$k = \frac{4}{3}$$

$$\operatorname{tg} \alpha' = \frac{3}{4} \implies \alpha' \approx 36^\circ 52'$$

$$\alpha = 90 - \alpha' \approx 53^\circ 8'$$