



Zadatak 55. U kojem su međusobnom odnosu kružnice:

- 1) $x^2 + y^2 - 10x + 6y + 30 = 0$ i
 $x^2 + y^2 + 2x - 2y - 47 = 0$;
- 2) $x^2 + y^2 - 6x + 10y + 18 = 0$ i
 $x^2 + y^2 + 6x - 6y - 18 = 0$;
- 3) $x^2 + y^2 + 2x - 6y - 15 = 0$ i
 $x^2 + y^2 - 4x + 2y - 4 = 0$;
- 4) $x^2 + y^2 - 2x + 4y + 4 = 0$ i
 $x^2 + y^2 + 4x - 4y - 28 = 0$?

Rješenje.

$$\begin{aligned} 1) \quad -10 &= -2p \implies p = 5 \\ 6 &= -2q \implies q = -3, S_1(5, -3) \\ 30 &= 25 + 9 - r^2 \implies r^2 = 4, r_1 = 2 \end{aligned}$$

$$\begin{aligned} 2 &= -2p \implies p = -1 \\ -2 &= -2q \implies q = 1, S_2(-1, 1) \\ -47 &= 1 + 1 - r^2 \implies r^2 = 49, r_2 = 7 \end{aligned}$$

$$\begin{aligned} |S_1 S_2| &= \sqrt{(-1 - 5)^2 + (1 + 3)^2} \\ &= \sqrt{36 + 16} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$$

$$r_1 + r_2 = 2 + 7 = 9$$

$$|S_1 S_2| < r_1 + r_2$$

Kružnice se sijeku u dvjema točkama.

$$\begin{aligned} 2) \quad -6 &= -2p \implies p = 3 \\ 10 &= -2q \implies q = -5, S_1(3, -5) \\ 18 &= 9 + 25 - r^2 \implies r^2 = 16, r_1 = 4 \end{aligned}$$

$$\begin{aligned} 6 &= -2p \implies p = -3 \\ -6 &= -2q \implies q = 3, S_2(-3, 3) \\ -18 &= 9 + 9 - r^2 \implies r^2 = 36, r_2 = 6 \end{aligned}$$

$$\begin{aligned} |S_1 S_2| &= \sqrt{(-3 - 3)^2 + (3 + 5)^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

$$r_1 + r_2 = 4 + 6 = 10$$

$$|S_1 S_2| = r_1 + r_2$$

Kružnice se diraju izvana.

$$\begin{aligned} 3) \quad 2 &= -2p \implies p = -1 \\ -6 &= -2q \implies q = 3, S_1(-1, 3) \\ 15 &= 1 + 9 - r^2 \implies r^2 = 25, r_1 = 5 \end{aligned}$$

$$\begin{aligned}
 -4 &= -2p \implies p = 2 \\
 2 &= -2q \implies q = -1, S_2(2, -1) \\
 -4 &= 4 + 1 - r^2 \implies r^2 = 9, r_2 = 3 \\
 |S_1 S_2| &= \sqrt{(2+1)^2 + (-1-3)^2} \\
 &= \sqrt{9+16} \\
 &= \sqrt{25} \\
 &= 5 \\
 r_1 + r_2 &= 5 + 3 = 8 \\
 |S_1 S_2| &< r_1 + r_2
 \end{aligned}$$

Kružnice se sijeku u dvjema točkama.

$$\begin{aligned}
 \mathbf{4)} \quad -2 &= -2p \implies p = 1 \\
 4 &= -2q \implies q = -2, S_1(1, -2) \\
 4 &= 1 + 4 - r^2 \implies r^2 = 1, r_1 = 1
 \end{aligned}$$

$$\begin{aligned}
 4 &= -2p \implies p = -2 \\
 -4 &= -2q \implies q = 2, S_2(-2, 2) \\
 -28 &= 4 + 4 - r^2 \implies r^2 = 36, r_2 = 6 \\
 |S_1 S_2| &= \sqrt{(-2-1)^2 + (2+2)^2} \\
 &= \sqrt{9+16} \\
 &= \sqrt{25} \\
 &= 5 \\
 |r_1 - r_2| &= |1 - 6| = 5 \\
 |r_1 - r_2| &= |S_1 S_2|
 \end{aligned}$$

Kružnice se diraju iznutra.