

Zadatak 58. Napiši jednadžbu kružnice koja prolazi točkama $A(3, -2)$ i $B(2, -1)$ i dira kružnicu $(x - 7)^2 + (y - 2)^2 = 16$.

Rješenje.

$$\begin{aligned} (3-p)^2 + (-2-q)^2 &= r^2 \\ (2-p)^2 + (-1-q)^2 &= r^2 \\ \hline 9 - 6p + p^2 + 4 + 4q + q^2 &= r^2 \\ 4 - 4p + p^2 + 1 + 2q + q^2 &= r^2 \\ \hline p^2 + q^2 - 6p + 4q + 13 &= r^2 \\ p^2 + q^2 - 4p + 2q + 5 &= r^2 \\ \hline -2p + 2q + 8 &= 0 \\ q &= p - 4 \\ p^2 + (p-4)^2 - 4p + 2(p-4) + 5 &= r^2 \\ p^2 + p^2 - 8p + 16 - 4p + 2p - 8 + 5 &= r^2 \\ 2p^2 - 10p + 13 &= r^2 \\ S_2(p, p-4) \end{aligned}$$

$$S_1(7, 2), r_2 = 4$$

Ako se kružnice diraju izvana vrijedi $r_1 + r = d(S, S_1)$

$$\begin{aligned} \sqrt{2p^2 - 10p + 13} + 4 &= \sqrt{(p-7)^2 + (p-4-2)^2}/2 \\ 2p^2 - 10p + 13 + 8\sqrt{2p^2 - 10p + 13} + 16 &= p^2 - 14p + 49 + p^2 - 12p + 36 \\ 8\sqrt{2p^2 - 10p + 13} &= -16p + 56 / : 8 \\ \sqrt{2p^2 - 10p + 13} &= 7 - 2p/2 \\ 2p^2 - 10p + 13 &= 49 - 28p + 4p^2 \\ -2p^2 + 18p - 36 &= 0 / : (-2) \\ p^2 - 9p + 18 &= 0 \\ p_{1,2} &= \frac{9 \pm \sqrt{81 - 72}}{2} = \frac{9 \pm 3}{2} \\ p_1 &= 6, \quad q_1 = 2 \\ p_2 &= 3, \quad q_2 = -1 \\ r^2 &= 2p^2 - 10p + 13 \\ r_1 &= 5, \quad r_2 = 1 \end{aligned}$$

Ako se kružnice diraju iznutra vrijedi $|r_2 - r| = d(S, S_2)$

$$\sqrt{2p^2 - 10p + 13} - 4 = \sqrt{(p-7)^2 + (p-4-2)^2}/2$$

$$2p^2 - 10p + 13 - 8\sqrt{2p^2 - 10p + 13} + 16 = p^2 - 14p + 49 + p^2 - 12p + 36$$

$$-8\sqrt{2p^2 - 10p + 13} = -16p + 56 / : (-8)$$

$$\sqrt{2p^2 - 10p + 13} = 2p - 7/2$$

$$2p^2 - 10p + 13 = 49 - 28p + 4p^2$$

$$-2p^2 + 18p - 36 = 0 / : (-2)$$

$$p^2 - 9p + 18 = 0$$

$$p_{1,2} = \frac{9 \pm \sqrt{81 - 72}}{2} = \frac{9 \pm 3}{2}$$

$$p_1 = 6, \quad q_1 = 2$$

$$p_2 = 3, \quad q_2 = -1$$

$$r^2 = 2p^2 - 10p + 13$$

$$r_1 = 5, \quad r_2 = 1$$

$$(x-3)^2 + (y+1)^2 = 1$$

$$(x-6)^2 + (y-2)^2 = 25.$$