Zadatak 59. Napiši jednadžbu kružnice koja prolazi točkama $A(1,1)$ i $B(0,2)$ i dira kružnicu
$(x-5)^2+(y-5)^2=16$.

Rješenje.

$$(1-p)^2+(1-q)^2=r^2$$
$$(0-p)^2+(2-q)^2=r^2$$

$$1-2p+p^2+1-2q+q^2=r^2$$
$$p^2+4-4q+q^2=r^2$$

$$p^2+q^2-2p-2q+2=r^2$$
$$p^2+q^2-4q+4=r^2$$

$$-2p-2q+4q-2=0$$
$$2q-2p=2$$
$$q-p=1 \implies q=p+1$$
$$p^2+(p+1)^2-4(p+1)+4=r^2$$
$$p^2+p^2+2p+1-4p-4+4=r^2$$
$$2p^2-2p+1=r^2$$
$$S_1(p,p+1)$$

$S(5,5)$, $r=4$
Ako se kružnice diraju izvana vrijedi $r_1+r=d(S,S_1)$

$$\sqrt{2p^2-2p+1}+4=\sqrt{(p-5)^2+(p+1-5)^2}/^2$$
$$2p^2-2p+1+8\sqrt{2p^2-2p+1}+16=p^2-10p+25+p^2-8p+16$$
$$8\sqrt{2p^2-2p+1}=-16p+24/:8$$
$$\sqrt{2p^2-2p+1}=3-2p/^2$$
$$2p^2-2p+1=9-12p+4p^2$$
$$-2p^2+10p-8=0/:(-2)$$
$$p^2-5p+4=0$$
$$p_{1,2}=\frac{5\pm\sqrt{25-16}}{2}=\frac{5\pm 3}{2}$$
$$p_1=4,\quad q_1=5$$
$$p_2=1,\quad q_2=2$$
$$r^2=2p^2-2p+1$$
$$r_1=5,\quad r_2=1$$

Ako se kružnice diraju iznutra vrijedi $|r_1 - r| = d(S, S_2)$

$$\sqrt{2p^2 - 2p + 1} - 4 = \sqrt{(p-5)^2 + (p+1-5)^2} /^2$$

$$2p^2 - 2p + 1 - 8\sqrt{2p^2 - 2p + 1} + 16 = p^2 - 10p + 25 + p^2 - 8p + 16$$

$$-8\sqrt{2p^2 - 2p + 1} = -16p + 24 / : (-8)$$

$$\sqrt{2p^2 - 2p + 1} = -3 + 2p /^2$$

$$2p^2 - 2p + 1 = 9 - 12p + 4p^2$$

$$-2p^2 + 10p - 8 = 0 / : (-2)$$

$$p^2 - 5p + 4 = 0$$

$$p_{1,2} = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2}$$

$$p_1 = 4, \quad q_1 = 5$$

$$p_2 = 1, \quad q_2 = 2$$

$$r^2 = 2p^2 - 2p + 1$$

$$r_1 = 5, \quad r_2 = 1$$

$(x-1)^2 + (y-2)^2 = 1\,,$
$(x-4)^2 + (y-5)^2 = 25\,.$