

Zadatak 15. Kružnica prolazi točkom $T(5, 1)$ i dira pravce $x - 2y + 6 = 0$ i $x - 2y - 4 = 0$. Kako glasi jednadžba te kružnice?

Rješenje. $y = \frac{1}{2}x + 3, \frac{1}{2}x - 2$

$$r^2 \left(1 + \frac{1}{4}\right) = \left(q - \frac{1}{2}p - 3\right)^2$$

$$r^2 \left(1 + \frac{1}{4}\right) = \left(q - \frac{1}{2}p + 2\right)^2$$

$$r^2 = (5 - p)^2 + (1 - q)^2$$

$$0 = \left(q - \frac{1}{2}p - 3\right)^2 - \left(q - \frac{1}{2}p + 2\right)^2$$

$$0 = \left(q - \frac{1}{2}p - 3 - q + \frac{1}{2}p - 2\right) \left(q - \frac{1}{2}p - 3 + q - \frac{1}{2}p + 2\right)$$

$$0 = -5(2q - p - 1) \implies p = 2q - 1$$

$$\frac{5}{4}r^2 = \left(q - q + \frac{1}{2} - 3\right)$$

$$\frac{5}{4}r^2 = \left(-\frac{5}{2}\right)^2$$

$$r^2 = 5$$

$$5 = (5 - 2q + 1)^2 + (1 - q)^2$$

$$5 = 36 - 24q + 4q^2 + 1 - 2q + q^2$$

$$5q^2 - 26q + 32 = 0$$

$$q_{1,2} = \frac{26 \pm \sqrt{676 - 640}}{10}$$

$$q_1 = 2, \quad p_1 = 3$$

$$q_2 = \frac{16}{5}, \quad p_2 = \frac{27}{5}$$

$$(x - 3)^2 + (y - 2)^2 = 5, \\ \left(x - \frac{27}{5}\right)^2 + \left(y - \frac{16}{5}\right)^2 = 5.$$