Zadatak 27. Pravci $x+2y+8=0$, $x-2y+8=0$ i $2x-y+1=0$ tangente su iste kružnice. Kako glasi njezina jednadžba?

Rješenje. Iz uvjeta da je središte $S(p,q)$ za r udaljeno od svakog od triju danih pravaca dobit ćemo sustav jednadžbi

$$\frac{|p+2q+8|}{\sqrt{1+4}}=r$$
$$\frac{|p-2q+8|}{\sqrt{1+4}}=r$$
$$\frac{|2p-q+1|}{\sqrt{4+1}}=r$$

$$(p+2q+8)^2=5r^2$$
$$(p-2q+8)^2=5r^2$$
$$(2p-q+1)^2=5r^2$$

$$(p+2q+8-p+2q-8)(p+2q+8+p-2q+8)=0$$
$$(p+2q+8-2p+q-1)(p+2q+8+2p-q+1)=0$$

$$4q(2p+16)=0$$
$$(-p+3q+7)(3p+q+9)=0$$

$$q_1=0 \implies (7-p)(3p+9)=0$$
$$p_{1,2}=\frac{4\pm\sqrt{16+84}}{2}=2\pm5$$
$$(-3+2\cdot0+8)^2=5r^2$$
$$25=5r^2 \implies r_1^2=5$$
$$(7+2\cdot0+8)^2=5r^2$$
$$225=5r^2 \implies r_2^2=45$$
$$2p+16=0 \implies p_3=-8$$
$$(8+3q+7)(-24+q+9)=0$$
$$q^2-10q-75=0$$
$$q_{3,4}=\frac{10\pm\sqrt{100+300}}{2}=5\pm10$$
$$(-8+2\cdot15+8)^2=5r^2$$
$$900=5r^2 \implies r_3^2=180$$
$$(-8+2\cdot(-5)+8)^2=5r^2$$
$$100=5r^2 \implies r_4^2=20$$

Postoje četiri rješenja: $(x+3)^2+y^2=5$, $(x-7)^2+y^2=45$, $(x+8)^2+(y-15)^2=180$, $(x+8)^2+(y+5)^2=20$.