Zadatak 38. Odredi jednadžbe zajedničkih tangenata dviju kružnica:

1) $(x+1)^2+y^2=5$ i $(x-4)^2+y^2=20$;

2) $(x-1)^2+(y+1)^2=25$ i
$(x-4)^2+(y-3)^2=100$;

3) $x^2+y^2-4x-2y+4=0$ i
$x^2+y^2+4x+2y-4=0$.

Rješenje. **1)** $S_1(-1,0)$, $r_1^2=5$
$S_2(4,0)$, $r_2^2=20$

$$5(1+k^2)=(0+k-l)^2$$
$$20(1+k^2)=(0-4k-l)^2$$
$$\hline$$
$$5+5k^2=k^2-2kl+l^2$$
$$20+20k^2=16k^2+8kl+l^2$$
$$\hline$$
$$4k^2+2kl-l^2=-5$$
$$4k^2-8kl-l^2=-20$$
$$\hline$$
$$10kl=15$$
$$k=\frac{3}{2l}\quad\frac{9}{l^2}+3-l^2=-5$$
$$l^4-8l^2-9=0$$
$$l_{1,2}^2=\frac{8\pm\sqrt{64+36}}{2}$$
$$l_{1,2}^2=\frac{8\pm 10}{2}$$
$$k_{1,2}^2=4\pm 5$$
$$l_{1,2}=\pm 3k_{1,2}=\pm\frac{1}{2}$$

$x-2y+6=0$ i $x+2y+6=0$.

2)

$$x^2 - 2x + 1 + y^2 + 2y + 1 = 25$$
$$x^2 - 8x + 16 + y^2 - 6y + 9 = 100$$

$$6x + 8y = -52$$
$$y = \frac{-26 - 3x}{4}$$
$$x^2 - 2x + \frac{(-26 - 3x)^2}{16} + 2 \cdot \frac{(-26 - 3x)}{4} = 23$$
$$16x^2 - 32x + 676 + 156x + 9x^2 - 208 - 24x = 368$$
$$25x^2 + 100x + 100 = 0$$
$$x^2 + 4x + 4 = 0$$
$$(x + 2)^2 = 0$$
$$x = -2$$
$$y = -5$$

Manja kružnica dira veću iznutra u točki $D(-2, -5)$.

$$(x_1 - p)(x - p) + (y_1 - q)(y - q) = r^2$$
$$(-2 - 1)(x - 1) + (-5 + 1)(y + 1) = 25$$

$$-3x + 3 - 4y - 4 = 25$$
$$3x + 4y + 26 = 0$$

Jedina zajednička tangenta je pravac $3x + 4y = -26$

3) $S_1(2,1)$, $r_1^2 = 1$
$S_2(-2,-1)$, $r_2^2 = 9$

$$(1+k^2) = (1-2k-l)^2$$
$$9(1+k^2) = (-1+2k-l)^2$$

$$1+k^2 = 1-4k+4k^2-2l+4kl+l^2$$
$$9+9k^2 = 4k^2-4k+1+2l-4kl+l^2$$

$$-3k^2-l^2+4k+2l-4kl = 0$$
$$5k^2-l^2+4k-2l+4kl+8 = 0$$

$$-8k^2-8 = -4l+8kl$$
$$-2k^2-2 = l(2k-1)$$
$$l = \frac{-2(k^2+1)}{2k-1}$$
$$l = \frac{2(k^2+1)}{1-2k}$$
$$-3k^2-\frac{4(k^2+1)^2}{(1-2k)^2}+4k+4\frac{k^2+1}{1-2k}\cdot(1-2k) = 0$$
$$-3k^2-4\frac{(k^2+1)^2}{(1-2k)^2}+4k+4k^2+4 = 0$$
$$k^2+4k+4 = 4\frac{(k^2+1)^2}{(1-2k)^2}$$
$$(k+2)^2 = \left(2\frac{k^2+1}{1-2k}\right)^2$$
$$k+2 = 2\frac{k^2+1}{1-2k}$$
$$k-2k^2+2-4k = 2k^2+2$$
$$-4k^2-3k = 0$$
$$k(4k+3) = 0$$
$$k_1 = 0, \quad l_1 = 2$$
$$4k = -3$$
$$k_2 = -\frac{3}{4} \quad l_2 = \frac{5}{4}$$

$y-2=0$,
$3x+4y-5=0$

$$k+2=-2\frac{k^2+1}{1-2k}$$
$$k-2k^2+2-4k=-2k^2-2$$
$$-3k=-4$$
$$k_3=\frac{4}{3},\quad l_3=-\frac{10}{3}$$

$4x-3y-10=0$
$x-1=0$