Zadatak 41. Odredi jednadžbe kružnica koje prolaze sjecištima pravca $x + y - 3 = 0$ i kružnice $(x-1)^2 + (y-2)^2 = 2$ i koje diraju pravac $y + 1 = 0$. Pod kojim se kutom sijeku te dvije kružnice?

Rješenje. $y = 3 - x$

$$\begin{aligned} x^2 - 2x + 1 + x^2 - 2x + 1 &= 2 \\ 2x^2 - 4x &= 0 \\ x(x-2) &= 0 \\ x_1 = 0, \quad y_1 &= 3 \\ x_2 = 2, \quad y_2 &= 1 \end{aligned}$$

$A(0,3)$, $B(2,1)$, $y = -1 \implies l = -1$

$$\begin{gathered} r^2(1+k^2) = (q - kp - l)^2 \\ r^2 = (q+1)^2 \\ (x-p)^2 + (y-q)^2 = r^2 \\ p^2 + (3-q)^2 = r^2 \\ (2-p)^2 + (1-q)^2 = r^2 \end{gathered}$$

$$\begin{gathered} p^2 + 9 - 6q + q^2 = r^2 \\ 4 - 4p + p^2 + 1 - 2q + q^2 = r^2 \end{gathered}$$

$$\begin{gathered} 4p - 4q + 4 = 0 \\ p = q - 1 \\ (q-1)^2 + 9 - 6q + q^2 = r^2 \\ q^2 - 2q + 1 + 9 - 6q + q^2 = (q+1)^2 \\ 2q^2 - 8q + 10 = q^2 + 2q + 1 \\ q^2 - 10q + 9 = 0 \\ q_{1,2} = \frac{10 \pm \sqrt{100 - 36}}{2} \\ q_1 = 9, \quad p_1 = 8, \quad r_1^2 = 100 \\ q_2 = 1, \quad p_1 = 0, \quad r_1^2 = 4 \end{gathered}$$

Uvjete zadatka zadovoljavaju dvije kružnice, $x^2 + (y-1)^2 = 4$ i $(x-8)^2 + (y-9)^2 = 100$.

$$(x_1 - p)(x - p) + (y_1 - q)(y - q) = r^2$$
$$(0 - 8)(x - 8) + (3 - 9)(y - 9) = 100$$
$$-8x + 64 - 6y + 54 = 100$$
$$y = -\frac{4}{3}x + 3$$
$$(0 - 0)(x - 0) + (3 - 1)(y - 1) = 4$$
$$2y - 2 = 4$$
$$y = 3$$

$$\operatorname{tg}\varphi = \left| \frac{0 + \frac{4}{3}}{1 - \frac{4}{3} \cdot 0} \right| = \frac{4}{3}.$$

$$\varphi = 53°08'.$$