

Zadatak 43. Pod kojim kutom dani pravac siječe dane kružnice?

- 1) $x + y - 7 = 0$, $x^2 + y^2 = 25$;
- 2) $x - 3y - 5 = 0$, $x^2 + y^2 - 2x + 6y + 5 = 0$;
- 3) $7x + 2y = 100$, $(x - 10)^2 + (y - 15)^2 = 45$;
- 4) $3x + y + 2 = 0$, $x^2 + y^2 - 4x + 6y - 12 = 0$.

Rješenje. 1) $y = 7 - x$

$$x^2 + 49 - 14x + x^2 = 25$$

$$2x^2 - 14x + 24 = 0$$

$$x^2 - 7x + 12 = 0$$

$$x_{1,2} = \frac{7 \pm \sqrt{49 - 48}}{2}$$

$$x_1 = 4, \quad y_1 = 3$$

$$x_2 = 3, \quad y_2 = 4$$

$$xx_1 + yy_1 = r^2$$

$$4x + 3y = 25$$

$$y = -\frac{4}{3}x + \frac{25}{3}$$

$$\operatorname{tg} \varphi = \left| \frac{-\frac{4}{3} + 1}{1 - \frac{4}{3} \cdot (-1)} \right| = \frac{1}{7}$$

$$\varphi = \operatorname{arc} \operatorname{tg} \frac{1}{7} \approx 8^\circ 8'$$

$$2) y = \frac{1}{3}x - \frac{5}{3}, \quad p = 1, \quad q = -3, \quad r^2 = 5$$

$$x^2 + \left(\frac{1}{3}x - \frac{5}{3}\right)^2 - 2x + 6\left(\frac{1}{3}x - \frac{5}{3}\right) + 5 = 0$$

$$x^2 + \frac{1}{9}x^2 - \frac{10}{9}x + \frac{25}{9} - 2x + 2x - 10 + 5 = 0$$

$$10x^2 - 10x - 20 = 0$$

$$x^2 - x - 2 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1 + 8}}{2}$$

$$x_1 = 2, \quad y_1 = -1$$

$$x_2 = -1, \quad y_2 = -2$$

$$(x_2 - p)(x - p) + (y_2 - q)(y - q) = r^2$$

$$(-1 - 1)(x - 1) + (-2 + 3)(y + 3) = 5$$

$$-2x + 2 + y + 3 = 5$$

$$y = 2x$$

$$\operatorname{tg} \varphi = \left| \frac{\frac{1}{3} - 2}{1 + 2 \cdot \frac{1}{3}} \right| = 1$$

$$\varphi = 45^\circ$$

3) $p = 10$, $q = 15$, $7 \cdot 10 + 2 \cdot 15 = 100 \implies$ pravac prolazi središtem kružnice pa je $\varphi = 90^\circ$.

4) $y = -3x - 2$, $p = 2$, $q = -3$, $r^2 = 25$

$$x^2 + (-3x - 2)^2 - 4x + 6(-3x - 2) - 12 = 0$$

$$x^2 + 9x^2 + 12x + 4 - 4x - 18x - 12 - 12 = 0$$

$$10x^2 - 10x - 20 = 0$$

$$x^2 - x - 2 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2}$$

$$x_1 = 2, \quad y_1 = -8$$

$$x_2 = -1, \quad y_2 = 1$$

$$(x_2 - p)(x - p) + (y_2 - q)(y - q) = r^2$$

$$(-1 - 2)(x - 2) + (1 + 3)(y + 3) = 25$$

$$-3x + 6 + 4y + 12 = 25$$

$$y = \frac{3}{4}x + \frac{7}{4}$$

$$\operatorname{tg} \varphi = \left| \frac{-3 - \frac{3}{4}}{1 - 3 \cdot \frac{3}{4}} \right| = 3$$

$$\varphi = \operatorname{arc} \operatorname{tg} 3 \approx 71^\circ 34'$$