

Zadatak 44. Pod kojim se kutom sijeku kružnice:

- 1) $(x - 2)^2 + (y - 2)^2 = 17$,
 $(x + 2)^2 + (y - 2)^2 = 1$;
- 2) $(x - 1)^2 + (y + 1)^2 = 18$,
 $(x - 2)^2 + (y + 2)^2 = 20$;
- 3) $x^2 + y^2 + 8x - 9 = 0$,
 $x^2 + y^2 + 9x - 7y + 20 = 0$;
- 4) $x^2 + y^2 = 5$, $x^2 + y^2 - 5x + 5 = 0$?

Rješenje. 1)

$$x^2 - 4x + 4 + y^2 - 4y + 4 = 17$$

$$x^2 + 4x + 4 + y^2 - 4y + 4 = 1$$

$$-8x = 16$$

$$x = -2$$

$$4 + 8 + 4 + y^2 - 4y + 4 = 17$$

$$y^2 - 4y + 3 = 0$$

$$y_{1,2} = \frac{4 \pm \sqrt{16 - 12}}{2} \quad y_1 = 1, \quad y_2 = 3$$

$$(x_1 - p)(x - p) + (y_1 - q)(y - q) = r^2$$

$$(-2 - 2)(x - 2) + (1 - 2)(y - 2) = 17$$

$$-4x + 8 - y + 2 = 17$$

$$y = -4x - 7$$

$$(-2 + 2)(x - 2) + (1 - 2)(y - 2) = 1$$

$$-y + 2 = 1$$

$$y = 1$$

$$\operatorname{tg} \varphi = \left| \frac{-4 + 0}{1 - 0 \cdot (-4)} \right| = 4$$

$$\varphi = \operatorname{arc} \operatorname{tg} 4 \approx 75^\circ 58'$$

2)

$$x^2 - 2x + 1 + y^2 + 2y + 1 = 18$$

$$x^2 - 4x + 4 + y^2 + 4y + 4 = 20$$

$$2x - 3 - 2y - 3 = -2$$

$$y = x - 2$$

$$x^2 - 2x + 1 + (x - 2)^2 + 2(x - 2) + 1 = 18$$

$$x^2 - 2x + x^2 - 4x + 4 + 2x - 4 = 16$$

$$2x^2 - 4x - 16 = 0$$

$$x^2 - 2x - 8 = 0$$

$$y_{1,2} = \frac{2 \pm 4 + 32}{2} x_1 = -2, \quad y_1 = -4$$

$$x_2 = 4, \quad y_2 = 2$$

$$(x_2 - p)(x - p) + (y_2 - q)(y - q) = r^2$$

$$(4 - 1)(x - 1) + (2 + 1)(y + 1) = 18$$

$$3x - 3 + 3y + 3 = 18$$

$$y = -x + 6$$

$$(4 - 2)(x - 2) + (2 + 2)(y + 2) = 20$$

$$2x - 4 + 4y + 8 = 20$$

$$y = -\frac{1}{2}x + 4$$

$$\operatorname{tg} \varphi = \left| \frac{-1 + \frac{1}{2}}{1 - \frac{1}{2} \cdot (-1)} \right| = \frac{1}{3}$$

$$\varphi = \operatorname{arc} \operatorname{tg} \frac{1}{3} \approx 18^\circ 26'$$

$$\begin{aligned} 3) \quad 8 &= -2p_1 \implies p_1 = -4, \quad q_1 = 0, \quad -9 = 16 - r_1^2 \implies r_1^2 = 25 \\ 9 &= -2p_2 \implies p_2 = -\frac{9}{2}, \quad -7 = -2q_2 \implies q_2 = \frac{7}{2}, \\ 20 &= \frac{81}{4} + \frac{49}{4} - r_2^2 \implies r_2^2 = \frac{25}{2} \end{aligned}$$

$$\begin{aligned} x^2 + 8x + y^2 - 9 &= 0 \\ x^2 + 9x + y^2 - 7y + 20 &= 0 \end{aligned}$$

$$-x + 7y - 29 = 0$$

$$x = 7y - 29$$

$$(7y - 29)^2 + 8(7y - 29) + y^2 - 9 = 0$$

$$49y^2 - 406y + 841 + y^2 + 56y - 232 - 9 = 0$$

$$50y^2 - 350y + 600 = 0$$

$$y^2 - 7y + 12 = 0$$

$$y_{1,2} = \frac{7 \pm \sqrt{49 - 48}}{2} \quad y_1 = 4, \quad x_1 = -1$$

$$y_2 = 3, \quad y_2 = -8$$

$$(x_1 - p)(x - p) + (y_1 - q)(y - q) = r^2$$

$$(-1 + 4)(x + 4) + (4 - 0)(y - 0) = 25$$

$$3x + 12 + 4y = 25$$

$$y = -\frac{3}{4}x + \frac{13}{4}$$

$$\left(-1 + \frac{9}{2}\right) \left(x + \frac{9}{2}\right) + \left(4 - \frac{7}{2}\right) \left(y - \frac{7}{2}\right) = \frac{25}{2}$$

$$\frac{7}{2}x + \frac{63}{4} + \frac{1}{2}y - \frac{1}{4} = 20$$

$$7x + y + 3 = 0$$

$$y = -7x - 3$$

$$\operatorname{tg} \varphi = \left| \frac{-7 + \frac{3}{4}}{1 + 7 \cdot \frac{3}{4}} \right| = 1$$

$$\varphi = 45^\circ$$

$$4) -5 = -2p_2 \implies p_2 = \frac{5}{2}, q_2 = 0, 5 = \frac{25}{4} - r_2^2 \implies r_1^2 = \frac{5}{4}$$

$$x^2 + y^2 = 5$$

$$x^2 + y^2 - 5x + 5 = 0$$

$$5x = 10$$

$$x = 2$$

$$4 + y^2 = 5$$

$$y^2 = 1$$

$$y_1 = 1, \quad y_2 = -1$$

$$xx_1 + yy_1 = r^2$$

$$2x + y = 5$$

$$y = -2x + 5(x_1 - p)(x - p) + (y_1 - q)(y - q) = r^2$$

$$\left(2 - \frac{5}{2}\right) \left(x - \frac{5}{2}\right) + (1 - 0)(y - 0) = \frac{5}{4}$$

$$-\frac{1}{2}x + \frac{5}{4} + y = \frac{5}{4}$$

$$y = \frac{1}{2}x$$

$$k_1 = -\frac{1}{k_2}$$

$$\varphi = 90^\circ.$$