Zadatak 11. Duljine stranica trokuta tri su uzastopna cijela broja, a najveći kut je dva puta veći od najmanjeg. Kolike su duljine stranica i koliki su kutovi trokuta?

Rješenje. Označimo sa $a = n$, $b = n+1$, $c = n+2$, $n \in \mathbf{N}$, duljine stranica trokuta. Tada je $\gamma = 2\alpha$, dakle imamo

$$a = a$$
$$b = a+1$$
$$c = a+2$$
$$\gamma = 2\alpha$$

$$a,\ b,\ c,\ \alpha,\ \beta,\ \gamma = ?$$

$$\frac{a}{a+2} = \frac{\sin\alpha}{\sin(2\alpha)}$$
$$\frac{a}{a+2} = \frac{\sin\alpha}{2\sin\alpha\cos\alpha}$$
$$\frac{a}{a+2} = \frac{1}{2\cos\alpha}$$
$$\cos\alpha = \frac{a+2}{2a}$$
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$a^2 = (a+1)^2 + (a+2)^2 - 2(a+1)(a+2)\cdot\frac{a+2}{2a}$$
$$a^2 = a^2 + 2a + 1 + a^2 + 4a + 4 - \frac{(2a+2)(a^2+4a+4)}{2a} \quad \Big/ \cdot (2a)$$
$$2a^3 = 4a^3 + 12a^2 + 10a - (2a^3 + 8a^2 + 8a + 2a^2 + 8a + 8)$$
$$2a^3 = 4a^3 + 12a^2 + 10a - 2a^3 - 10a^2 - 16a - 8$$
$$2a^2 - 6a - 8 = 0$$
$$a^2 - 3a - 4 = 0$$
$$a^2 + a - 4a - 4 = 0$$
$$a(a+1) - 4(a+1) = 0$$
$$(a+1)(a-4) = 0$$
$$a > 0 \implies a+1 > 0 \implies a-4 = 0 \implies a = 4, \quad b = 5, \quad c = 6$$
$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc} = \frac{25+36-16}{2\cdot5\cdot6} = \frac{45}{60} = \frac{3}{4}, \quad \alpha = 41°24'33''$$
$$\cos\beta = \frac{a^2+c^2-b^2}{2ac} = \frac{16+36-25}{2\cdot4\cdot6} = \frac{27}{48} = \frac{9}{16}, \quad \beta = 55°46'21''$$
$$\gamma = 180° - 41°24'33'' - 55°46'21'' = 82°49'6''.$$