Zadatak 3. Ne rabeći računalo izračunaj površinu trokuta $\triangle ABC$ ako je:

1) $b=\sqrt{6}$, $c=\sqrt{3}(1+\sqrt{3})$, $\alpha=45^\circ$;

2) $a=1+\sqrt{3}$, $\beta=45^\circ$, $\gamma=60^\circ$;

3) $b=2$, $c=\sqrt{3}-1$, $\beta=135^\circ$;

4) $a=1+\sqrt{3}$, $b=\sqrt{6}$, $c=\sqrt{3}-1$.

Rješenje. **1)**

$$b=\sqrt{6}$$
$$c=\sqrt{3}(1+\sqrt{3})$$
$$\alpha=45^\circ$$

$$P=?$$

$$P=\frac{b\cdot c\cdot\sin\alpha}{2}=\frac{\sqrt{6}\cdot\sqrt{3}(1+\sqrt{3})\cdot\frac{\sqrt{2}}{2}}{2}=\frac{\sqrt{36}(1+\sqrt{3})}{4}=\frac{3}{2}(1+\sqrt{3})$$

2)

$$a=1+\sqrt{3}$$
$$\beta=45^\circ$$
$$\gamma=60^\circ$$

$$P=?$$
$$\alpha=180^\circ-\beta-\gamma=75^\circ$$
$$P=\frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$$
$$\sin 75^\circ=\sin(45^\circ+30^\circ)=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$$
$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{2}(\sqrt{3}+1)}{4}$$
$$P=\frac{(1+\sqrt{3})^2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}(\sqrt{3}+1)}{4}}=\frac{(1+\sqrt{3})\sqrt{3}}{2}.$$

3) $b = 2$

$c = \sqrt{3} - 1$

$\beta = 135^\circ$

$P = ?$

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\sin\gamma = \frac{c\sin\beta}{b} = \frac{(\sqrt{3}-1)\sin 135^\circ}{2} = \frac{(\sqrt{3}-1)\frac{\sqrt{2}}{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4} = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \sin 60^\circ\cos 45^\circ - \cos 60^\circ \sin 45^\circ = \sin(60^\circ - 45^\circ) = \sin 15^\circ$$

$\gamma = 15^\circ$

$\alpha = 180^\circ - \beta - \gamma = 180^\circ - 135^\circ - 15^\circ = 30^\circ$

$$P = \frac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \frac{4\cdot\frac{1}{2}\cdot\frac{\sqrt{6}-\sqrt{2}}{4}}{2\cdot\frac{\sqrt{2}}{2}} = \frac{1}{2}(\sqrt{3}-1).$$

4) $a = 1 + \sqrt{3}$

$b = \sqrt{6}$

$c = \sqrt{3} - 1$

$P = ?$

$$s = \frac{a+b+c}{2} = \frac{1+\sqrt{3}+\sqrt{6}+\sqrt{3}-1}{2} = \frac{2\sqrt{3}+\sqrt{6}}{2} = \sqrt{3} + \frac{\sqrt{6}}{2}$$

$$P = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{\left(\sqrt{3}+\frac{\sqrt{6}}{2}\right)\left(\sqrt{3}+\frac{\sqrt{6}}{2}-1-\sqrt{3}\right)\left(\sqrt{3}+\frac{\sqrt{6}}{2}-\sqrt{6}\right)\left(\sqrt{3}+\frac{\sqrt{6}}{2}-\sqrt{3}+1\right)}$$
$$= \sqrt{\left(\sqrt{3}+\frac{\sqrt{6}}{2}\right)\left(\frac{\sqrt{6}}{2}-1\right)\left(\sqrt{3}-\frac{\sqrt{6}}{2}\right)\left(\frac{\sqrt{6}}{2}+1\right)}$$
$$= \sqrt{\left(\sqrt{3}+\frac{\sqrt{6}}{2}\right)\left(\sqrt{3}-\frac{\sqrt{6}}{2}\right)\left(\frac{\sqrt{6}}{2}-1\right)\left(\frac{\sqrt{6}}{2}+1\right)}$$
$$= \sqrt{\left(3-\frac{3}{2}\right)\left(\frac{3}{2}-1\right)} = \sqrt{\frac{3}{2}\cdot\frac{1}{2}} = \frac{\sqrt{3}}{2}.$$